DESCRIPTION OF THE PROJECT NEAR-RINGS WITH RIGHT IDENTITY

1. Zusammenfassung

Eines der Themen mit der sich die Algebra befasst ist das Rechnen mit Objekten wie wenn es Zahlen wären. Kann man die Objekte addieren und subtrahieren, so spricht man von Gruppe. Kann man auch multiplizieren wie wir es von den ganzen Zahlen gewohnt sind, so entsteht ein Ring. Wenn man zwar multiplizieren aber Klammerausdrücke nur von rechts ausmultiplizieren kann, so nennt man die Struktur einen Fastring. Hat man ein Objekt, das sich beim Multiplizieren wie eine Eins benimmt, so spricht man von einem Fastring mit Einselement. Diese Fastringe kennt man gut. Sie lassen sich mit sogenannten Zentralisatorfastringen beschreiben. Dabei handelt es sich um eine Menge von Funktionen von einer Gruppe in sich selbst. Die Addition erfolgt wie üblich bei Funktionen und die Multiplikation ist die Hintereinanderausführung der Funktionen. Um alle Fastringe mit Eins zu erhalten darf man aber nicht immer alle Funktionen der Gruppe in sich selbst nehmen, sondern man muss geschickt auswählen. Dies geschieht durch die Zentralisatoreigenschaft. Fastringe die man nicht so gut kennt sind jene, die keine multiplikative Eins besitzen. Diese Fastringe können sehr interessant sein, da sie unübliche Methoden des Rechnens erlauben. Tatsächlich haben viele interessante Fastringe keine Eins, aber immer noch so etwas wie eine halbe Eins. Bei Multiplikation von rechts benimmt sich diese halbe Eins wie eine Eins, bei Multiplikation von Links aber nicht. Dies hat oft erstaunliche Konsequenzen was das Rechnen innerhalb dieser Strukturen betrifft. Besonders interessante Klassen von Fastringen sind Fastringe genau solchen Typs, zum Beispiel planare Fastringe - mit einer Vielzahl von Anwendungen - oder primitive Fastringe, die kleinsten Bausteine aus denen Fastringe zusammengesetzt sind. Trotzdem sind Fastringe mit einer halben Eins bis heute wenig untersucht worden, es fehlten effiziente Methoden zu deren Untersuchung. Mit Hilfe von Funktionen auf Gruppen die nicht mehr auf die ganze Gruppe selbst abbilden, sondern nur auf eine echte Teilmenge davon, lassen sich alle Fastringe mit einer halben Eins konstruieren. Dazu muss eine neue Multiplikation, die sogenannte Sandwich - Multiplikation, eingeführt und mit der Zentralisatoreigenschaft kombiniert werden. Mit Hilfe dieser Methode die der Autor in seiner Dissertation entwickelt und in weiterführenden Publikationen angewandt hat, soll in diesem Projekt eine systematische Untersuchung von Fastringen mit einer halben Eins erfolgen.

2. Project Summary

One of the topics algebra is concerned with is computing with objects as if they were numbers. If these objects can be added and subtracted one speaks of a group. If furthermore one can multiply these objects as one is used with the integers one speaks of a ring. If one still can multiply, but can multiply out brackets only from the right hand side and not from the left hand side, one speaks of a near-ring. If there is an object which under multiplication behaves like an identity 1, then one speaks of a near-ring with identity. These near-rings can be fully described as functions mapping from a group into itself. Addition is the usual addition of functions and multiplication is function composition. If one wants to get all near-rings with identity that way, one cannot always take all functions mapping from the group into itself. One has to select functions in a clever way. This is done by the so called centralizer property. Near-rings one does not know so good are near-rings which do not have an identity. These near-rings can be very interesting because they allow unusual methods of computing. Most of these near-rings still have something like a half sided identity. This element behaves like an identity when it is multiplied from the right hand side but not when it is multiplied from the left hand side. Often, this results in interesting methods of computing. In particular, many interesting classes of near-rings are of that type, for example planar near-rings - with a lot of applications inside and outside of algebra - and primitive near-rings, the smallest building stones any near-ring is built of in some sense. Nevertheless, there is no systematic study of near-rings with only a half sided identity up to now. One reason for that is that there has not been an efficient method to describe them so far. With the help of functions mapping from a group only into a subset of this group and together with a new multiplication, the so called sandwich multiplication combined with the centralizer property, one can get an efficient method to describe all near-rings with a half sided identity. This method was developed in the author' dissertation and was successfully used in some forthcoming papers. However, a deeper research in that direction has not been done so far. This will be done in this project.

3. Scientific Aspects

The scientific work will take place at the Institute of Algebra at the Johannes Kepler Universität in Linz, Austria. This institute, formed around Prof. Günter Pilz, is one of the leading institutes in research concerning near-rings.
A good motivation for studying near-rings is the fact that the set of all functions on a group G forms a near-ring w.r.t. pointwise addition of functions and function composition. These functions do not have to be linear and also the additive group G does not have to be abelian, so we get something more general than a ring.

Definition 3.1. A (right) near-ring is a set N together with two binary operations "+" and "$*$" such that

(1) $(N,+)$ is a group (not necessarily abelian).
(2) $(N,*)$ is a semigroup.
(3) $\forall n_1,n_2,n_3 \in N:\ (n_1+n_2)*n_3 = n_1*n_3+n_2*n_3$.

If in addition we have that $n*0=0$ for all $n \in N$, then the near-ring N is called zero symmetric. Very basic examples of near-rings are the following:
Let $M(\Gamma) := \{f:\Gamma \longrightarrow \Gamma\}$ be the set of all functions from the group $(\Gamma,+)$ into itself. Then $(M(\Gamma),+,\circ)$ is a right near-ring. Let $M_0(\Gamma) := \{f:\Gamma \longrightarrow \Gamma | f(0)=0\}$ where 0 is the zero of the group $(\Gamma,+)$. Then $(M_0(\Gamma),+,\circ)$ is a zero symmetric near-ring. $M(\Gamma)$ as well as $M_0(\Gamma)$ contain the identity function, so they are examples of near-rings with identity. In some sense, all of the near-rings with identity arise as some kind of near-rings of functions, the so called centralizer near-rings.

Definition 3.2. Let $(\Gamma,+)$ be a group and let $\varnothing \neq S \subseteq \mathrm{End}(\Gamma)$. Define $M_S(\Gamma) := \{f:\Gamma \longrightarrow \Gamma \mid \forall s \in S: f\circ s = s\circ f\}$. Then $M_S(\Gamma)$ is a near-ring under function composition and pointwise function addition. We call such a near-ring a centralizer near-ring.

Theorem 3.3 ([8], Theorem 2.8)**.** *Every zero symmetric near-ring with identity is (isomorphic to) a centralizer near-ring $M_S(\Gamma)$, for a suitable group Γ and $S \subseteq$* $\mathrm{End}(\Gamma)$.

In fact S can be taken as subsemigroup of $\mathrm{End}(\Gamma)$, since $M_S(\Gamma) = M_{<S>}(\Gamma)$, where $<S>$ is the semigroup generated by S.
Much research concerning near-rings with identity has been made in the light of Theorem 3.3. For example, one may consider special semigroups S or consider special conditions on orbits of S on Γ. The additive group Γ may also be of a special nature.
However, there do exist interesting types of near-rings which do not have an identity. Therefore many of the standard knowledge one has about near-rings with identity does not carry over to them. One type of near-rings without identity which in recent research has raised interest (see [9], [6], [4], [11], [12], [13], [7]) are the planar near-rings.

Planar near-rings are a class of near-rings which have a lot of applications. On one hand, they have real life applications because they can be used to construct highly efficient balanced incomplete block designs (see [9], [7], [10]). On the other hand, they have a strong impact on the structure theory of near-rings (see [11], [12]). However, in the moment a planar near-ring has an identity, it is a (planar) near-field and a lot of its structural richness which enables applications goes lost. It does not seem to be an exaggeration to say that planar near-rings have so many interesting and important properties because they lack an identity element. A similar situation occurs when considering primitive near-rings. 1- and 2-primitive near-rings with an identity element can be described completely as centralizer near-rings, where S is a subgroup of the automorphism group of Γ which acts without fixed points (see [9]). If we consider primitive near-rings without identity, then the situation immediately gets much more difficult. In [13] 1-primitive near-rings with a half sided identity have been characterized using so called sandwich centralizer near-rings which we will introduce in a moment.

It seems that the absence of an identity element in a near-ring can give a lot of interesting structural properties to the near-ring. So far, there is no deeper investigation in near-ring theory which especially focuses on that fact. Interestingly, whenever a finite zero symmetric near-ring N has an element which is not a zero divisor it has a so called right identity element 1_r. This means that $n * 1_r = n$ for all $n \in N$ but not necessarily $1_r * n = n$ (see [9]). So, a lot of near-rings still have a half sided identity. There is a method to construct all such zero symmetric near-rings, using a more general concept than centralizer near-rings. Basically, the multiplication has to be twisted.

The following construction of certain types of sandwich near-rings was introduced by the author in his PHD thesis and applied in some forthcoming papers ([4], [11], [13]).

Let $(N,+)$ be a group, $X \subseteq N$ a subset of N containing the zero 0 of $(N,+)$ and $\phi : N \longrightarrow X$ a map such that $\phi(0) = 0$. Define the following operation $\circ'$ on N^X (N^X denoting the set of all functions mapping from X to N): $f \circ' g := f \circ \phi \circ g$ for $f, g \in N^X$. Then $(N^X, +, \circ')$ is a near-ring, denoted by $M(X,N,\phi)$.

Let $S \subseteq \mathrm{End}(N,+)$, S not empty, be such that $\forall s \in S, n \in N :\ \phi \circ s(n) = s \circ \phi(n)$ and such that $S(X) \subseteq X$. Then $M_0(X,N,\phi,S) := \{ f : X \longrightarrow N \mid f(0) = 0 \text{ and } \forall s \in S, x \in X : f(s(x)) = s(f(x))\}$ is a zero symmetric subnear-ring of $M(X,N,\phi)$, which we call a sandwich centralizer near-ring.

The concept of sandwich centralizer near-rings turned out to be of great use: Any zero symmetric near-ring with a multiplicative right identity is isomorphic to a sandwich centralizer near-ring. This is a generalization of the well known result that any zero symmetric near-ring with identity is a centralizer near-ring.

Theorem 3.4. [4] *Let M be a near-ring. Then the following are equivalent:*

(1) *M is a zero symmetric near-ring with right identity.*

(2) *There exists a group $(N,+)$ and a subset X of N with $0 \in X$, there exists a non-empty subset $S \subseteq \mathrm{End}(N,+)$ with $S(X) \subseteq X$, and there exists a function $\phi : N \longrightarrow X$ with $\phi(0) = 0$, $\phi\mid_X = id$ and $\phi \circ s(n) = s \circ \phi(n)$ for all $s \in S$ and $n \in N$, such that $M \cong M_0(X,N,\phi,S)$.*

Again, S can be taken as subsemigroup of $\mathrm{End}(N,+)$. Note that if $\phi = id$ and $X = \Gamma$, then $M_0(X,N,\phi,S)$ will be the zero symmetric part of the usual centralizer near-ring $M_S(N)$.
As already mentioned, in [13] a similar theorem was proved for 1-primitive near-rings having a multiplicative right identity and in [4] a similar theorem has been proved for planar near-rings. Basically, one has to model the input parameters X,N,ϕ,S to get those near-rings.

4. Goals of the project

4.1. **Interesting classes of near-rings.** Any zero symmetric near-ring with a right identity can be described as sandwich centralizer near-ring $M_0(X,N,\phi,S)$. It would be interesting to see what kind of near-rings come out by modelling the input parameters X,N,ϕ,S. A deeper research in that direction has not been done so far, to the author's knowledge. Only for planar and primitive near-rings (see [13], [4]) the method was used so far, and it was used with great success. Deeper structural questions could be considered using the method of sandwich centralizer multiplication (see again [13], [4] and also [12] for details).
Certainly, the number of near-rings with a half sided identity is much too big to consider all of them. So, I would start my research with very basic questions conserning the input parameters X,N,ϕ,S. It seems to be interesting to look at the following questions and see what comes out:

(1) What happens if X is a subgroup of Γ?
(2) What comes out if ϕ is a homomorphism or has other special properties?
(3) What happens if we focus on special classes of groups N?
(4) What happens if S is a semigroup of endomorphisms of special type, for example S is a group, or a regular semigroup?
(5) What happens if we put conditions on orbits of S on N and on X, for example conditions on the number of orbits, disjoint orbits, stabilizer conditions and so on? (This sort of question already showed up in [13], [4], so here we would have a link between older and new research.)

This list of questions should point out in which direction research in this context could go on. The hope would be to get interesting classes of near-rings, interesting in that sense that they may have a strong impact on the structure theory of near-rings, as planar or primitive near-rings have, and also interesting in the sense that they have applications, like planar near-rings have. A first step in that direction could be to model the input parameters giving primitive or planar near-rings (see [4], [11], [13]).
Since dealing with sandwich centralizer near-rings is a very wide field, I would also put a focus on what questions have been considered when studying centralizer near-rings. This may give ideas how interesting questions (as mentioned above) can be attacked.

4.2. **Multiplicative semigroups.** Let N be a right near-ring. For $u \in N$ let $\psi_u : N \longrightarrow N, n \mapsto n * u$ be the right translation map induced by u. Note that ψ_u is an endomorphism of $(N,+)$. For a subsemigroup $S \subseteq N$ of the multiplicative semigroup of the near-ring we define $\Psi_S := \{\psi_s \mid s \in S\}$. Therefore, $(\Psi_U, \circ)$ is a semigroup of endomorphisms of $(N,+)$. In particular, we can let $S = N$. If the near-ring N has an identity element, then via the function $\phi : N \longrightarrow \Psi_N, n \mapsto \phi_n$ the semigroups

$(N,*)$ and $(\Psi_N,\circ)$ turn out to be anti-isomorphic. This is no longer the case when N has no identity element anymore. ϕ only is anti-epimorphic in such a case. Especially if N has a multiplicative right identity 1_r, then ψ_{1_r} is the identity function. So $(\Psi_N,\circ)$ has an identity element and is a monoid but $(N,*)$ has no identity element. The multiplicative semigroup of a near-ring has been studied in [14] from a very general point of view. Also the book [1] has a focus on the multiplicative structure of near-rings. Nevertheless, the study of near-rings where $(N,+)$ is a group of special type is much more developped (see [9]) as the study of near-rings where $(N,*)$ is a semigroup of special type. In particular, to study near-rings where $(\Psi_N,\circ)$ is of special type could be interesting for near-rings with a half sided identity, since $(\Psi_N,\circ)$ and $(N,*)$ are non-anti - isomorphic. The semigroup $(\Psi_N,\circ)$ and the semigroup S of endomorphisms when considering sandwich centralizer near-rings are closely related. In fact, when shaping a near-ring into a sandwich centralizer near-ring, Ψ_N will take the role of S (see [4]). Also in case of planar near-rings it is the semigroup $(\Psi_N,\circ)$ which makes those near-rings so special (see [9], [6]). We give a list of questions that seem interesting to study:

(1) Study $(\Psi_N,\circ)$ in its own right using methods of semigroup theory and try to find an impact to the structure theory of near-rings. For example, what can be said about Green's relations in $(\Psi_N,\circ)$? Do they have an impact on the structure of the near-ring?
(2) Are there any deeper relations between $(\Psi_N,\circ)$ and $(N,*)$ in case N has no identity element? For example, in case N has no identity, there are no invertible elements in N. If N has a multiplicative right identity, $(\Psi_N,\circ)$ is a monoid. So there exist $n\in N$ giving invertible functions in Ψ_N. Do these elements n play a specific role in the structure of the near-ring? They would be something like units in N, but N has no identity. Certainly, these elements are multiplicatively closed. What happens if they are also additively closed, so form an algebraic substructure? What kind of impact would this have on the structure of the parent near-ring? More general, one can raise questions like this: Given a semigroup substructure of special type in $(\Psi_N,\circ)$ or $(N,*)$, does this have an impact on the structure of the near-ring?

4.3. **Near-rings without a right identity.** In [2] the authors describe 1-primitive near-rings not even having a multiplicative right identity. The authors use so-called centralizer sandwich near-rings which seem to be closely related to our sandwich centralizer near-rings, which however have a multiplicative right identity. It would be interesting to see connections between both constructions which could lead to an efficient method to describe more classes of near-rings without even having a multiplicative right identity.

4.4. **Application of planar near-rings.** I worked part time in project P 19463 N 18 of the Austrian National Science Fund. During this project applications of planar near-rings to agricultural questions where made (see [10]). This research was carried out together with the Landwirtschaftskammer (probably best translated as chamber of agriculture) Oberösterreich. If there is new request from the Landwirtschaftskammer or if there would be other chances to carry out these methods of applying abstract algebra, then the author would like to take the chance to do this. The methods how to do this are known and do not have to be newly invented, they only have to be adapted for new situations.

5. Possible further consequences

As pointed out, near-rings without an identity, especially planar near-rings, have many applications to other fields in mathematics, for example statistics or geometry. This also leads to real life applications (see the section above). Probably other classes of near-rings without an identity which are similar to planar near-rings have also similar applications.

6. Methods and timetable, financial aspects

Some of the methods how I plan my research work have already been implicitely mentioned. As already pointed out, the project work will be carried out at the Institute of Algebra at the Johannes Kepler Universität in Linz, Austria. There I find all the resources I need to do mathematical research at modern level. Hence, it is not necessary for me to apply for further financial resources, other than my salary.

I plan to be embedded in the research work people are doing at this institute. Several world leading experts in near-ring theory, especially the head of the department Prof. Günter Pilz, do research there. Especially, it is planned to attend the weekly research seminars held at the institute, to exchange and discuss mathematical ideas. Apart from standard techniques of doing mathematical research, the use of the computer algebra system SONATA (see [5]), developped at the Institute of Algebra at the Johannes Kepler Universität, is planned. Another focus will be on a close contact and maybe scientific collaboration with the near-ring research group at the National Cheng Kung University in Tainan / Taiwan around Prof. Wen Fong Ke.

Furthermore, I plan to attend international conferences on algebra to discuss and spread scientific results, in the same spirit as I have done during the last years as project member of FWF projects.

Concerning the time table, I will start my research with 4.1 and 4.2. Since the multiplicative semigroup of the near-ring as well as the semigroup of endomorphisms S play a role there, 4.1 and 4.2 will be considered in parallel. I plan to spend a year and a half on that question. As a next step, in the following one and a half year, one can try to generalize results as mentionned in 4.3 and look for applications as pointed out in 4.4.

7. Personal career development

I have already worked in FWF projects for 6 years. This project could give me the chance to stay within the scientific community in Austria. Since I will work part time as a high school teacher in Austria, I do not have the chance to go abroad and apply for other types of scientific jobs. Also, I consider the link between being a scientist and working as a high school teacher to be interesting. This could also be interesting in a future educational system in Austria.

8. Scientific career and list of Publications

I was born on July 25, 1977 in Steyr, Austria. I am Austrian citizen and live in 4400 St. Ulrich, Finkenweg 7. After finishing high school in 1995 I started studying mathematics and physics at the Johannes Kepler University in Linz for becoming a teacher (Lehramtsstudium Mathematik und Physik). I finished in July 2001. After that I started a PhD study for mathematics, also at the Johannes Kepler University in Linz. I finished in June 2004. In September 2004 I started teaching in high schools in Austria, what I still do. Since September 2002, I am also teaching at the Johanes Kepler University (with a break in the years 2007 and 2008). I did research work in two projects funded by the FWF (details see below) in the years 2002 to 2005 and 2007 to 2010, but not in a full time position. My main research topic is Algebra, especially abstract algebra with a focus on near-rings. I will also give a short overview concerning my career in the following:

Education: Doktor der Naturwissenschaften, June 2004, Ph.D. thesis *Planarity in Near-rings* supervised by Prof. Dr. Günter Pilz, Linz University.

Magister der Naturwissenschaften, July 2001, Diploma thesis *Zahlensysteme* supervised by Prof. Dr. Günter Pilz, Linz University.

I was finishing high school (Matura) in 1995.

Experience: Participation in two research projects on near-rings funded by the FWF, lead by Prof. Dr. Günter Pilz.

Planar Near-rings: Theory and Applications, P 15691 (from the year 2002 to 2005), and Polynomial Functions on Omega-groups P 19463 N18 (from the year 2007 to 2010). In the project P 19463 N18 I was in a Postdoc position.

Only from May 2002 until September 2004 I was in a full research position in the FWF projects. For the rest of the time I basically was in a half time position, some months even less.

This is because I am teaching in high schools in Austria since September 2004. Currently I am teaching at the HTL Grieskirchen (HTL stands for *höhere technische Lehranstalt* - something like a school for higher technical education at pre-university level).

Also I am teaching at Johannes Kepler University in Linz since 2002 up to now with a break in the years 2007 and 2008.

Presentations relevant to the project at international conferences in Hamburg, Potsdam, Klagenfurt 2003, Taiwan 2004 and 2005, Linz and Darmstadt 2007, Linz 2008 and Vorau (Austria) 2009.

Publications:

The following list covers the publications of the last five years. Only in the years 2007 to 2010 I was working in a scientific position.

1. Gerhard Wendt, "Minimal Left Ideals of Near-rings" , *Acta Math. Hungar.*, Vol. 127(1-2), doi: 10.1007/s10474-010-9090-1, 52-63, 2010
2. Gerhard Wendt, Johan H. Meyer, Wen-Fong Ke, "Matrix maps over planar near-rings" , *Proceedings of the Royal Society of Edinburgh: Section A Mathematics*, doi: 10.1017/S0308210508000899, Vol. 140A, 83-99, 2010
3. Gerhard Wendt, "On zero divisors in near-rings" , *International Journal of Algebra*, Vol. 3, no. 1, 21-32, 2009
   (online: http://www.m-hikari.com/ija/index.html)
4. Gerhard Wendt, "Primitive Near-rings - Some Structure Theorems" , *Algebra Colloquium*, 14:3, 417-424, 2007

The other publications including the dissertation of the author are the following:

5. Gerhard Wendt, ”Left ideals in 1-primitive near-rings” , *Mathematica Pannonica*, Vol. 16(1), 145-151, 2005
6. Gerhard Wendt, ”On the number of zero divisors in near-rings” , *Contributions to General Algebra 16*, 289-291, 2005
7. Gerhard Wendt, ”Planar Near-rings, Sandwich Near-rings and Near-Rings with Right Identity” , *Nearrings and Nearfields*, 277–291, Springer, 2005
8. Gerhard Wendt, ”On the multiplicative semigroup of near-rings” , *Mathematica Pannonica* Vol. 15(2), 2004
9. Gerhard Wendt, ”Characterisation Results for Planar Near-Rings” , *Contributions to General Algebra*, Vol. 15, 2004
10. Gerhard Wendt, ”Planarity in Near-Rings” , Dissertation, Universität Linz, 6-2004

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