1^{st} practice sheet multivariate methods II

Helmut Waldl

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1. Show: in product-multivariate schemata we have the following equivalence:

$$(p_{1j} = \cdots = p_{Ij} \qquad j = 1, \ldots, J) \quad \Longleftrightarrow$$
$$\Longleftrightarrow \quad (m_{ij} = \frac{m_{i+} \cdot m_{+j}}{m_{++}} \qquad i = 1, \ldots, I \quad j = 1, \ldots, J)$$

2. Show: in Poisson schemata we have the following equivalence:

$$\mu_{ij} = \frac{\mu_{i+} \cdot \mu_{+j}}{\mu_{++}} \quad \Longleftrightarrow \quad \tilde{m}_{ij} = \frac{\tilde{m}_{i+} \cdot \tilde{m}_{+j}}{\tilde{m}_{++}} \qquad i = 1, \ldots, I \quad j = 1, \ldots, J$$

   where: $\tilde{m}_{ij} = E(x_{ij}|x_{++})$.

3. Prove the following theorem from our lecture:

   The hypothesis $m_{ij} = \frac{m_{i+} \cdot m_{+j}}{m_{++}}$ is equivalent to the log-linear independence model for the

   (a) Poisson schema

   (b) multinomial schema with side condition

$$x_{++} = \sum_{i,j} \exp(\mu + \mu_{A(i)} + \mu_{B(j)})$$

   (c) product multinomial schema with side conditions

$$x_{i+} = \sum_{j} \exp(\mu + \mu_{A(i)} + \mu_{B(j)}) \qquad i = 1 \ldots, I$$

4. Given a multiplicative Poisson-model in the variates A and B. The values of A are in $\{1, \ldots, I\}$, the values of B in $\{1, \ldots, J\}$. Estimating the model parameters we get the ML-equations:

$$\hat{m}_{++} = x_{++} \qquad \hat{m}_{i+} = x_{i+} \qquad \hat{m}_{+j} = x_{+j}$$

   Show: Solving the ML-equations for $\hat{m}_{ij}$ considering the conditions of the independence model $m_{ij} = \frac{m_{i+} \cdot m_{+j}}{m_{++}}$ we get the ML-estimators

$$\hat{m}_{ij} = \frac{x_{i+} \cdot x_{+j}}{x_{++}} \qquad \hat{\mu} = \frac{1}{I} \sum_{i} \ln(x_{i+}) + \frac{1}{J} \sum_{j} \ln(x_{+j}) - \ln(x_{++})$$

$$\hat{\mu}_{A(i)} = \ln(x_{i+}) - \frac{1}{I} \sum_{i} \ln(x_{i+}) \qquad \hat{\mu}_{B(j)} = \ln(x_{+j}) - \frac{1}{J} \sum_{j} \ln(x_{+j})$$