# Parameter estimation in a spatial autoregressive model

Sándor Baran

Department of Probability Theory and Applied Mathematics
Faculty of Informatics
University of Debrecen, Hungary

We consider the spatial autoregressive process $\{X_{k,\ell} : k, \ell \geq 0\}$ defined as

$$X_{k,\ell} = \alpha X_{k-1,\ell} + \beta X_{k,\ell-1} + \gamma X_{k-1,\ell-1} + \varepsilon_{k,\ell},$$

for $k, \ell \geq 1$, and $0$ otherwise, where the independent innovations $\varepsilon_{k,\ell}$ have zero mean and unit variance. The model is stable inside a tetrahedron with vertices $(1, 1, -1)$, $(1, -1, 1)$, $(-1, 1, 1)$ and $(-1, -1, -1)$ and unstable on the boundary of this domain (Basu & Reinsel, 1993). We investigate the asymptotic properties of the least squares estimator (LSE) of the parameters in the unstable case.

In the special case $\gamma = 0$ Paulauskas (2007) determined the asymptotic behaviour of the variances of the process and Baran *et al.* (2007) showed that in the unstable case the LSE of $(\alpha, \beta)$ is asymptotically normal and the rate of convergence is $n^{-3/2}$ if one of the parameters equals zero and $n$ otherwise.

In the general model the limiting behaviour of the variances of the process and the behaviour of the LSE also depend on the location of the parameters (Baran, 2011). We show that the limiting distribution of the LSE of the parameters $(\alpha, \beta, \gamma)$ is normal and the rate of convergence is $n$ when the parameters are in the faces or on the edges of the boundary of the domain of stability, while on the vertices the rate is $n^{3/2}$.