Merged: Spatial Chow-Lin methods for data completion in econometric flow models

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Research questions

- Data on low regional scales are often incomplete
- Chow and Lin (1971) data completion for time series
- Develop a **spatial** Chow-Lin method for missing spatial data
- Maximum Likelihood and Bayesian estimation for spatial Chow-Lin
- Compare the spatial predictions to the naive predictions
- outline assumptions and sensitivity checks

Chow-Lin steps

Spatial Chow-Lin (CL) in 4 steps

- 1. Specify disaggregate model $y_d = X_d\beta + \epsilon$, y_d is not, X_d is observed
- 2. With known aggregation matrix C get the agg. model $y_a = Cy_d = X_a\beta + \epsilon_a$, y_a, X_a is observed
- aggregate model $Cy_d = CX_d\beta + C\epsilon$,
- 3. GLS estimation: $\hat{\beta} = (X_a'(C\Omega C')^{-1}X_a)^{-1}X_a'(C\Omega C')^{-1}y_a$
- 4. Conditional prediction of disaggregate y: $\hat{y}_d = X_d\hat{\beta} + \Omega C'(C\Omega C')^{-1}(y_a - X_a\hat{\beta}) = Naive + Gain * Agg.error$ (in the joint model of the aggregates and disaggregates)
- The covariance matrix of the aggregate errors is model dependent.

Spatial Chow-Lin LS Estimator: SAR model

Structural SAR Model:

$$y_d = \rho_d W_N y_d + X_d \beta_d + \epsilon_d, \qquad \epsilon_d \sim \mathcal{N}[0, \sigma_d^2 I_N]. \tag{1}$$

Reduced form:

$$y_d = R_N^{-1} X_d \beta_d + R_N^{-1} \epsilon_d, \quad R_N^{-1} \epsilon_d \sim \mathcal{N}[0, \Sigma_d]. \tag{2}$$

VC matrix:

$$\Sigma_d = \sigma_d^2 (R_N' R_N)^{-1}, \tag{3}$$

Aggregated reduced form:

$$C y_d = C R_N^{-1} X_d \beta + C R_N^{-1} \epsilon, \quad C R_N^{-1} \epsilon \sim N[0, \Sigma_a]. \tag{4}$$

Spatial Chow-Lin (SCL) Estimator

GLS estimator:

$$\hat{\beta}_{GLS} = (X_d' C' (C \hat{\Sigma}_d C')^{-1} C X_d)^{-1} X_d' C' (C \hat{\Sigma}_d C')^{-1} y_a \quad (5)$$

Prediction in the disaggregate model

$$\hat{y}_d = \hat{R}_N^{-1} X_d \hat{\beta}_{GLS} + \hat{\Sigma}_d C' (C \hat{\Sigma}_d C')^{-1} (y_a - C \hat{R}_N^{-1} X_d \hat{\beta}_{GLS}). \quad (6)$$

whereas the spatial improvement of the Goldberger (1962) 'gain projection matrix' $G = \hat{\Sigma}_d C' (C \hat{\Sigma}_d C')^{-1}$ distributes the aggregate residuals $(y_a - C \hat{R}_N^{-1} X_d \hat{\beta}_{GLS})$ to the spatial naive prediction $(\hat{R}_N^{-1} X_d \hat{\beta}_{GLS})$.

Estimate of ρ_d

To perform the spatial Chow-Lin we need an estimate of ρ_d. For simplicity we use the estimate at the aggregate regional level:

$$y_a = \rho_a W_n y_a + C X_d \beta + \nu_a, \quad \nu_a \sim \mathcal{N}[0, \sigma_a^2 I_N], \tag{7}$$

which is used to construct the disaggregate covariance matrix Σ_d from equation (3).

Chow-Lin assumptions

Assumptions

1. Structural similarity: $\hat{y}_d = f(\hat{\beta}_a, \hat{\Sigma}_d)$ where $\hat{\Sigma}_d = f(\hat{\rho}_a)$
2. Error similarity: $\hat{\rho}_a \approx \hat{\rho}_d$ whereas W_d and W_a are commensurable for aggregation
3. Reliability of indicators: adequate R^2 for the dis-/aggregate model

Bayesian Spatial Chow-Lin (BSCL) Estimation

The prior distribution for the parameters $\theta_a = (\beta_a, \sigma_a^{-2}, \rho_a)$ is proportional to

$$
\begin{aligned}
p(\theta_a) &\propto p(\beta_a) \cdot p(\sigma_a^{-2}) && (8) \\
&= \mathcal{N}[\beta_a \mid b_*, H_*] \cdot \Gamma(\sigma_a^{-2} \mid s_*^2, n_*), && (9)
\end{aligned}
$$

whereas we assume $\rho_a \sim U[-1, 1]$, and the joint distribution is given by

$$
p(\theta, y_a) = \mathcal{N}[CR_N^{-1} X_d \beta, C\Sigma_d C'] \mathcal{N}[\beta \mid b_*, H_*] \cdot \Gamma(\sigma^{-2} \mid s_*^2, n_*) \quad (10)
$$

MCMC in the SAR-CL model

Theorem (MCMC in the SAR-CL model)

Repeat the following steps until convergence:

1. *Draw* β *from* $\mathcal{N}\left[\beta \mid b_{**}, H_{**}\right]$
2. *Draw* ρ_i *by a Metropolis step:* $\rho_{new} = \rho_{old} + \mathcal{N}(0, \tau^2)$
3. *Draw* σ^{-2} *from* $\Gamma[\sigma^{-2} \mid s^2_{**}, n_{**}]$

We receive the MCMC posterior sample of θ_a

$$\Theta_{MCMC} = \{(\beta_j, \rho_j, \sigma_j^2), \quad j = 1, ..., J\}. \tag{11}$$

SAR-CL prediction for iteration j

$$y_d^{(j)} \sim \mathcal{N}[R_d^{(j)-1} X \beta^{(j)} + g^{(j)}, \sigma^{2(j)}[(R_d^{(j)\prime} R_d^{(j)})^{-1} - G^{(j)}]], \quad (12)$$

with the gain vector that shifts the conditional mean

$$g^{(j)} = (R_d^{(j)\prime} R_d^{(j)})^{-1} C' [C (R_d^{(j)\prime} R_d^{(j)})^{-1} C']^{-1} (y_a - \hat{y}_a^{(j)}), \quad (13)$$

and the gain matrix

$$G^{(j)} = (R_d^{(j)\prime} R_N^{(j)})^{-1} C' [C (R_d^{(j)\prime} R_d^{(j)})^{-1} C']^{-1} C (R_d^{(j)\prime} R_d^{(j)})^{-1}, \quad (14)$$

that reduces the unconditional covariance matrix.

Application to EU-27 NUTS 2 GDP

- dependent variable: GDP at NUTS 1 level (84 regions, Eurostat)
- independent variable: Employment (emp) at NUTS 2 level (239 regions, Eurostat)
- 24 EU-27 countries, some regions dropped due to changing classification
- W is the inverse car travel time matrix (map24.com) - threshold 180 minutes

Applications

To predict GDP (and Globalisation) at NUTS 2 level

compare the

- spatial Chow-Lin (SCL) predictions (gain term with: $\Sigma_d = \sigma_d^2(R_N' R_N)^{-1}$) to
- the pro rata (PR) prediction (gain term with: $\Sigma_d = I_d$)
- by means of forecast criteria based on the comparison with the disaggregated observed dependent variable (NUTS 2 GDP that is actually available)
- also compare ML to Bayesian forecasts

Regression comparison: NUTS 1 and NUTS 2

Table: Regression results, Dependent Variable GDP, 2005

	NUTS 1		NUTS 2	
Variable	ML	Bayesian	ML	Bayesian
c	-15131.14***	-11602.45***	-22680.26***	-13442.97***
	(3585.58)	(3674.09)	2969,40	(2172.2)
emp	50.82***	49.71***	60.04***	52.9***
	(4.24)	(4.26)	(2.2)	(1.98)
$W^{180} * GDP$	0.36***	0.32***	0.25***	0.19***
	(0.05)	(0.05)	(0.04)	(0.03)
R^2	0.7223	0.7288	0.7262	0.7271
Observations	84	84	239	239

***(**)[*] indicates that the coefficient is significant at a 1% (5%) [10%] level.

Prediction Accuracy

As the assumptions are fulfilled, we observe an improvement in the forecast criteria between 3 and 28%. Between ML and Bayesian, the forecasts improve by between 5% to 43%.

Table: Ratio of forecast criteria

	RMSE	MAE	MAPE
pro rata\SCL			
ML	1.14	1.12	1.28
Bayesian	1.03	1.04	1.17
ML\Bayesian			
pro rata gain term	1.16	1.13	1.43
spatial gain term	1.05	1.05	1.31

Sensitivity Analysis

- sensitivity check of the assumptions by predicting series from different specifications
- example: model with spatial threshold of 90 minutes
- the spatial parameter ρ shows large deviations between the aggregate (~ 0.25) and disaggregate (~ 0.10) estimation
- assumption 2 is violated

Prediction Accuracy: violation of assumption 2

We observe higher forecasting errors (between 1 and 18%) using the spatial Chow-Lin method, as assumption 2 is violated.

Table: Ratio of forecast criteria, threshold = 90 minutes

	RMSE	MAE	MAPE
pro rata\SCL			
ML	0.93	0.95	0.99
Bayesian	0.82	0.89	0.95
ML\Bayesian			
pro rata gain term	1.07	1.07	1.31
spatial gain term	0. 95	1.00	1.26

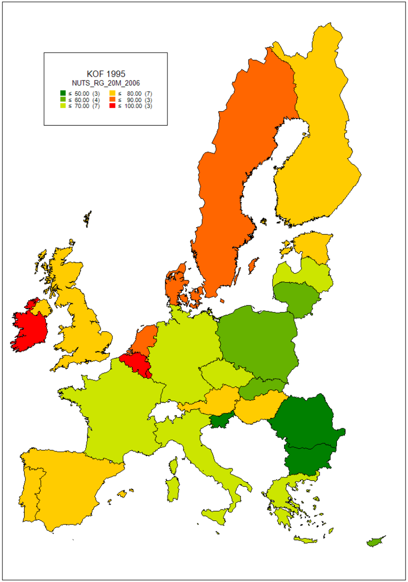
Conclusions and outlook

- The spatial Chow-Lin (SCL) method can be applied to disaggregated regional cross-section data
- SCL improves prediction when compared to naive (pro rata) forecast
- Bayesian SCL methods further improves the prediction compared to the ML estimation
- SCL works also for panel data and panel flow models (paper Polasek et al. 2010)
- SCL methods can be used for even smaller units like NUTS 3, cities
- Assumptions for SCL are more crucial than for time series Chow-Lin

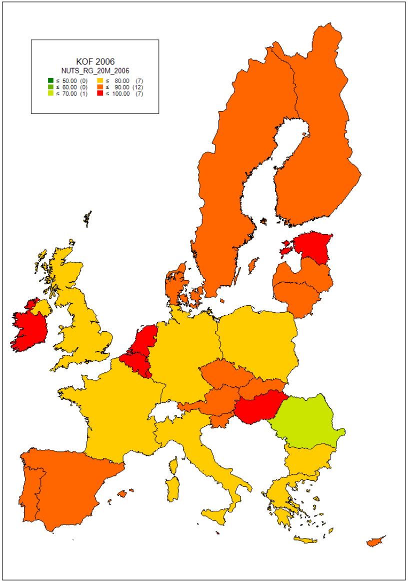
Economic and Social Globalization Index of ETH

- A. Economic Globalization
- i) Data on actual Flows
- Trade (percent of GDP)
- Foreign Direct Investment, flows (percent of GDP)
- Foreign Direct Investment, stocks (percent of GDP)
- Portfolio Investment (percent of GDP)
- Income Payments to Foreign Nationals (percent of GDP)
- ii) Data on restrictions
- Hidden Import Barriers
- Mean Tariff Rate
- Taxes on International Trade (percent of current revenue)
- Capital Account Restrictions
- B. Social Globalization
- i) Data on Personal Contact ii) Data on Information Flows iii) Data on Cultural Proximity
- C. Political Globalization

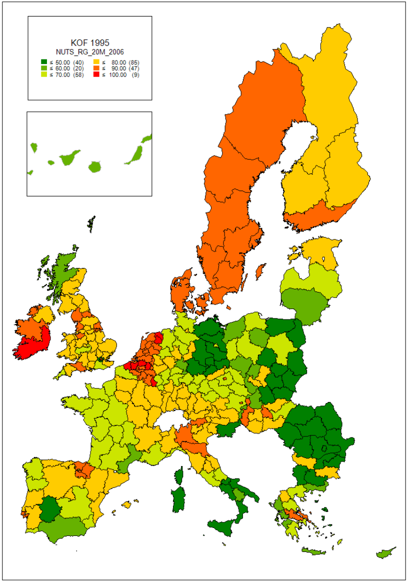
Globalisation Indices NUTS0



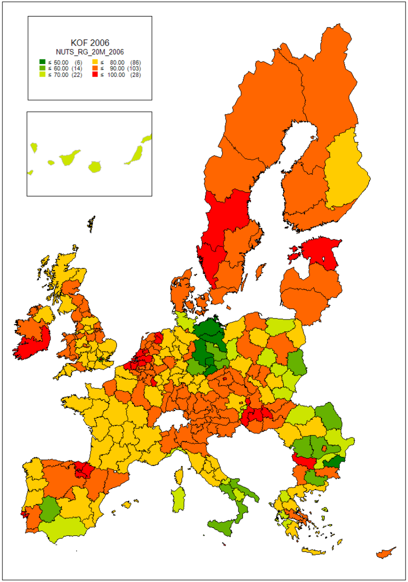
Globalisation Indices 2006 NUTS0



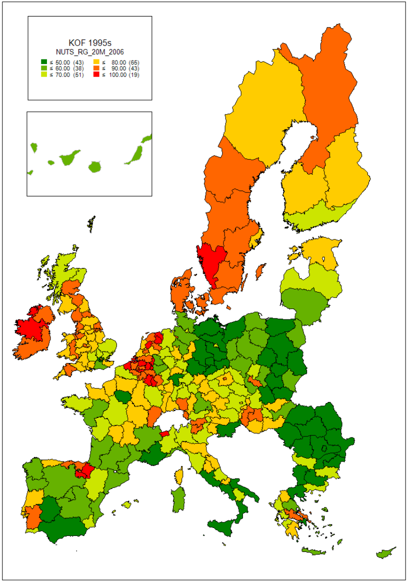
Regional Globalisation Indices 1995 NUTS2



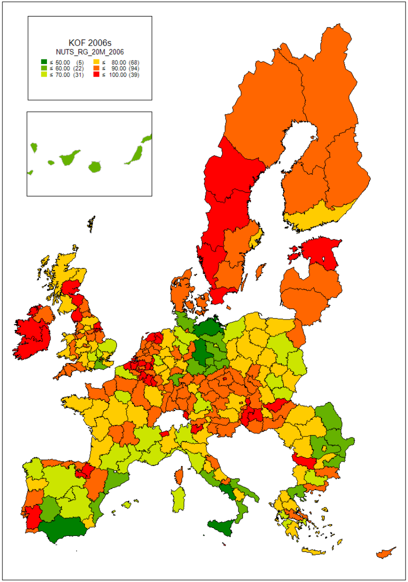
Regional Globalisation Indices 2006 NUTS2



SAR Globalisation Indices 1995



SAR Globalisation Indices 2006



OLS Chow-Lin Estimates Indices 2006

```
Ordinary Least-squares Estimates
Dependent Variable = log odds KOF ECO
R-squared      =    0.6927
Rbar-squared   =    0.6785
sigma^2        =    0.1828
Durbin-Watson  =    1.8524
Nobs, Nvars    =    228,    11
***********************************************************
Variable                          Coefficient     t-statistic    t-prob.
c                                    15.004          6.445        0.000
log GDPpc                             0.797          4.176        0.000
log GVA share market services        -2.067         -4.346        0.000
log ALP market services              -0.392         -1.937        0.054
log popdens                           0.072          1.526        0.128
log KVA                              -0.549         -4.299        0.000
log household expenditure per gdp    -2.002         -6.623        0.000
log retail spending per gdp           0.271          1.848        0.065
log unemployment rate                -0.306         -3.984        0.000
log Active to Pop ratio              -2.370         -6.085        0.000
log investment rate                   0.711          3.867        0.000
```

SAR Chow-Lin Estimates Indices 2006

```
Pooled model with spatial lag dependent variable, no fixed effects
Dependent Variable = log odds KOF ECO
R-squared            =    0.7490
Rbar-squared         =    0.7362
sigma^2              =    0.1421
Nobs,Nvar,TNvar      =    228,    11,    12
log-likelihood       =   -101.93146
# of iterations      =    16
min and max rho      =   -1.0000,   1.0000
total time in secs =    0.0280
time for lndet       =    0.0190
time for t-stats     =    0.0010
Pace and Barry, 1999 MC lndet approx. used
order for MC appr    =     50
iter  for MC appr    =     16
***************************************************************
Variable                          Coefficient  Asy. t-stat    z-probability
c                                    13.273        6.169        0.000
log GDPpc                             0.732        4.307        0.000
log GVA share market services        -1.565       -3.500        0.000
log ALP market services              -0.396       -2.217        0.026
log popdens                           0.008        0.177        0.858
log KVA                              -0.418       -3.505        0.000
log household exp. per gdp           -1.922       -7.070        0.000
log retail spendings per gdp          0.271        2.096        0.036
log unemployment rate                -0.301       -4.431        0.000
log Active to Pop ratio              -2.403       -6.997        0.000
log investment rate                   0.537        3.147        0.001
rho                                   0.340        2.279        0.022
```

Overview

- The Chow-Lin method is popular for completing data in disaggregated time series problems.
- This paper will do an extension of this approach:
- Spatial Internal Flow (SIF) models with Chow-Lin.
- Empirical Application EU27 Trade Flows
- Conclusions

Assumptions

Assumption

Structural similarity: The aggregated model for $\mathbf{y}_c$ and the disaggregated model for $\mathbf{y}$ are structurally similar. This implies that variable relationships that are observed on an aggregated level are following the same empirical law as on a disaggregated level: the regression parameters in both models are the same.

Assumption

Error similarity: The spatially correlated errors have a similar error structure on an aggregated level and on a disaggregated level: The spatial correlations are not significantly different.

Assumption

Reliable indicators: The indicators to make the formats on a disaggregated level have sufficiently large predictive power: The R^2 (or the F test) is significantly different from zero.

Notation in the spatial internal flow (SIF) model

Let $Y_a : N \times N$ be the aggregate panel matrix
T aggregated time points and N aggregate cross-sectional units
$Y : n \times n$ be the disaggregate panel matrix. Note: aggregation has to be done from both sides

$$Y_a = C_0 Y C_0'.$$

The aggregation matrix is $C_0 = C_0 : N \times n$ with $n > N$ across space faces irregularities and is defined as a block diagonal matrix:

$$C_0 = diag(1_{n_1}, ..., 1_{n_N})$$

where the $n_i's$ are the lengths of $1_{n_i} : n_i \times 1$.

Aggregated model

Vectorize the aggregation equation of the flows:

$$y_a = (C_0 \otimes C_0)y = Cy \tag{15}$$

joint aggregation matrix $C = C_0 \otimes C_0$,
panel regressors $X : m \times n$ ... " indicator"',
vectorized: $vecX = x : TM \times 1$.
The disaggregated model is a regression model on vectorized panel matrices:

$$y = X\beta + \epsilon, \quad \epsilon \sim N[0, \Omega \otimes \sigma^2 V] \tag{16}$$

where V and Ω are $n \times n$.
Simpler assumption: $\epsilon \sim N[0, \sigma^2 I_{nm}]$.

spatial lag polynomial
We assume a model of the for LeSage and Pace(2006):

$$y = \rho(W_1, W_2)y + X\beta + u,$$

where $\rho(W_1, W_2)y$ stands for a spatial lag polynomial that is applicable for flow models

$$\rho(W_1, W_2)y = \rho_1(W_1 \otimes I_n)y + \rho_2(I_n \otimes W_2)y + \rho_3(W_1 \otimes W_2)y$$

The aggregated reduced form (ARF)
Multiply the reduced form by the aggregation matrix C:

$$\mathbf{Cy} = \rho(\mathbf{W_1}, \mathbf{W_2})\mathbf{y} + \mathbf{CR}_\rho^{-1}\mathbf{X}\beta + \mathbf{CR}_\rho^{-1}\mathbf{u},$$

where the spread matrix $\mathbf{R}_\rho$ for flows is given by

$$\mathbf{R}_\rho = I_{nn} - \rho_1(W_1 \otimes I_n) + \rho_2(I_m \otimes W_2) + \rho_3(W_1 \otimes W_2)$$

and W_1 and W_2 are suitable chosen neighborhood matrices

SIF: RF and ARF

The reduced form (RF) of the spatial internal flow model is

$$y = \tilde{R}^{-1}X\beta + \tilde{R}^{-1}\epsilon, \quad \tilde{R}^{-1}\epsilon \sim N[0, \sigma^2 V_\rho] \tag{17}$$

with $V_\rho = \tilde{R}^{-1}(\Omega \otimes V)\tilde{R'}^{-1}$.

For the estimation we need the aggregated reduced form ARF

$$Cy = C\tilde{R}^{-1}X\beta + C\tilde{R}^{-1}\epsilon, \quad C\tilde{R}^{-1}\epsilon \sim N[0, C\tilde{R}^{-1}(\Omega \otimes \sigma^2 V)\tilde{R}'^{-1}C']. (18)$$

ARF of the SIF model

Only the aggregated data are completely observed: derive the aggregated model from the disaggregated model in RF. The ARF model (18) is ($C : N \times n$):

$$y_a = X_{a\rho}\beta + \epsilon_{a\rho}, \quad \epsilon_{a\rho} \sim N[0, \sigma^2 V_{a\rho}]. \tag{19}$$

with

$y_a = Cy$, $X_{a\rho} = C\tilde{R}^{-1}X$,

and

$$V_{a\rho} = C\tilde{R}^{-1}(\Omega \otimes V)\tilde{R}'^{-1}C'. \tag{20}$$

Lag polynomials for flows

The general spatial lag polynomial is

$$\mathbf{R}_\rho = I_{nn} - \rho_1(W_1 \otimes I_n) - \rho_2(I_n \otimes W_2) + \rho_3(W_1 \otimes W_2) = \quad (21)$$

$$= \rho_1\tilde{\mathbf{R}}_1 + \rho_2\tilde{\mathbf{R}}_2 - \rho_3\tilde{\mathbf{R}}_3 \quad (22)$$

with the following 3 components

$$\tilde{\mathbf{R}}_1 = I_{nn} - \rho_1(W_1 \otimes I_n) = R_1 \otimes I_n$$

$$\tilde{\mathbf{R}}_2 = I_{nn} - \rho_2(I_n \otimes W_2) = I_n \otimes R_2$$

$$\tilde{\mathbf{R}}_3 = I_{nn} - \rho_3(W_1 \otimes W_2) = R_1 \otimes R_2$$

and $R_i = I_n - \rho_i W_i, i = 1, 2.$

Instead of assuming the whole polynomial we could estimate the components individually.

The spatial $W \otimes W$-SIF flow model
Spatial inetraction in the flow model (16)

$$y = \rho(W_2 \otimes W_1)y + X\beta + \epsilon, \quad \epsilon \sim N[0, \Omega \otimes \sigma^2 V]. \tag{23}$$

or

$$y = \rho\tilde{y} + X\beta + \epsilon,$$

spatial lag vector $\tilde{y}$, a special "interaction"
between the neighborhoods of the origin and destination regions:

$$\tilde{y} = vec(W_1 Y W_2') = (W_2 \otimes W_1)y.$$

The reduced form

is given by

$$\tilde{R}y = X\beta + \epsilon, \quad \epsilon \sim N[0, \sigma^2 \Omega \otimes V]. \tag{24}$$

Thus the reduced and aggregated reduced form are in this special case formally the same as in the general case (17) and (18).

Special Case

The reduced form of the spatial flow model is for the cases (1) and (2)

$$y = \tilde{R}^{-1} X\beta + \tilde{R}^{-1}\epsilon, \quad \tilde{R}^{-1}\epsilon \sim N[0, \sigma^2 \Omega_\rho \otimes V_\rho]. \tag{25}$$

with

$$\Omega_\rho = (R_2' \Omega^{-1} R_2)^{-1} \quad and \quad V_\rho = (R_1' V^{-1} R_1)^{-1}$$

with $R_i = I_n - \rho W_i, i = 1, 2$ alternating taking the paired possibilities (W_2, I_n) or (I_n, W_1).

The ARF model

This ARF model is the starting point. Assume K panel indicators: $X_1, \ldots, X_K$,
where the first one is a matrix of ones $X_1 = E_{NN} = 1_N \otimes 1'_N$.
Define the regressor matrix of vectorized panels

$$X = (vecX_1, \ldots, vecX_K) : (NN \times K) \tag{26}$$

...

$$CX = X_a : (nn \times K) \tag{27}$$

Now the aggregated model is obtained by multiplying with the aggregation matrix C but now the aggregation matrix is given by:

$$\begin{aligned} X_a &= (vecC1_n1'_nC', vec(CX_2C'), ..., vec(CX_KC')) = \\ &= (vecX_{a_1}, vecX_{a_2}, ..., vecX_{a_K}). \end{aligned}$$

Stability

The transposed matrix X_a is $K \times nn$ and given by

$$\begin{pmatrix} vec' X_{a_1} \\ .. \\ vec' X_{a_K} \end{pmatrix} \tag{28}$$

Note: A 2-step estimate is given like in the simple spatial Chow-Lin model (see Polasek 2008).

First, we consider the usual GLS estimate $\beta_{GLS} : (K \times 1)$:

$$\beta_{GLS} = (X_a' V_{a\rho}^{-1} X_a)^{-1} (X_a' V_{a\rho}^{-1} y_a),$$

and then the "weighted" GLS estimate

$$\beta_{WLS} = (X_a' V_{a\rho}^{-1} X_a)^{-1} (X_a' V_{a\rho}^{-1}) \tilde{y}_a$$

using the spatial lag $\tilde{y}_a = vec W_1 Y_a W_2'$ as dependent variable.

The GLS and WLS estimates in the SIF model

$$\beta_{GLS} = M_{\tilde{X}\tilde{X}}^{-1} M_{\tilde{X}\tilde{Y}} \tag{29}$$

$$\beta_{WLS} = M_{\tilde{X}\tilde{X}}^{-1} M_{\tilde{X}\tilde{Y}W'} \tag{30}$$

are $K \times 1$ estimated vectors. $M_{\tilde{X}\tilde{X}}$ and $M_{\tilde{X}\tilde{Y}}$ are cross-moments matrices
of the aggregated and transformed observations $\tilde{X}$ and $\tilde{Y}$ respectively.

Proof:

$$M_{\tilde{X}\tilde{X}} = \left(\begin{pmatrix} vec' X_{a_1} \\ ... \\ vec' X_{a_K} \end{pmatrix} (\Omega_a \otimes V_a)^{-1} X_a \right) =$$

$$\begin{pmatrix} trV_a^{-1} X_{a1} \Omega_a^{-1} X'_{a1} & ... & V_a^{-1} X_{aK} \Omega_a^{-1} X'_{a1} \\ ... & ... & \\ trV_a^{-1} X_{a1} \Omega_a^{-1} X'_{aK} & ... & V_a^{-1} X_{aK} \Omega_a^{-1} X'_{aK} \end{pmatrix}$$

using

$$trABCD = vec' D' (C' \otimes A) vecB$$

and $vec' D = (vecD)'$ denotes the row vectorization.

The cross-moment matrix

In the same way we find for the $(K \times K)$ cross-moment matrix

$$M_{\tilde{X}\tilde{Y}} = \left(X_a'(\Omega_a \otimes V_a)^{-1} vec(Y_a)\right) = \begin{pmatrix} trV_a^{-1} Y_a \Omega_a^{-1} X_{a1}' \\ \dots \\ trV_a^{-1} Y_a \Omega_a^{-1} X_{aK}' \end{pmatrix}$$

and

$$M_{\tilde{X}\tilde{Y}} = \left(X_a'(\Omega_a \otimes V_a)^{-1} vec(W_1 Y_a W_2')\right) = \begin{pmatrix} trV_a^{-1} W_1 Y_a W_2' \Omega_a^{-1} X_{a1}' \\ \dots \\ trV_a^{-1} W_1 Y_a W_2' \Omega_a^{-1} X_{aK}' \end{pmatrix}$$

Spatial rho estimation

The minimum of the spatial ρ is found by minimizing the error sum of squares (ESS):

$$ESS_{\rho,\phi} = (e_0 - \rho e_W)'(e_0 - \rho e_W) \qquad (31)$$

with $e_0 = y_a - X\beta_{GLS}$ and $e_W = y_a - X\beta_{WLS}$.
Note that the minimum of ρ can be found by obtaining the feasible root of the quadratic equation in (31):

$$ESS(\rho) = q_2\rho^2 - q_1\rho + q_0$$

with $q_2 = e_a'e_a, q_1 = 2e_W'e_a$ and $q_0 = e_W'e_W$.
For the estimation of the residual variance-covariance matrices we define the matrices

$$\hat{\Omega}_a = E_a'E_a/N, \quad \hat{V}_a = E_aE_a'/N$$

. For the 4-step procedure we repeat now the previous step but replace Ω_a^{-1} and V_a^{-1} by their estimates $(\hat{\Omega})_a^{-1}$ and $(\hat{V}_a)^{-1}$. Clearly, this can be repeated until convergence.

GLS point prediction

The forecasting has to be done by the Goldberger (1962) formula:

$$\hat{y} = X\beta_{GLS} + G\hat{e}$$

where the $G\hat{e}$ is an improvement of the estimated error term $\hat{e} = (y_a - X_{a\rho}\beta_{GLS})$ using the "Goldberger gain" matrix

$$G = V_{a\rho}^{-1}C'(CV_{a\rho}^{-1}C')^{-1}.$$

The forecasts can be calculated as

$$\hat{y} = X\beta_{GLS} + G\hat{e} \tag{32}$$

and the matrix is obtained by de-vectorizing: $\hat{Y} = vec\hat{y}$.

Empirical Application

We predict the origin/destination flows of 259 NUTS-2 regions for a sample of the 27 EU (NUTS-0) countries for 2006 by a classical trade gravity model. As explaining factors, we use the population and GDP from the origin and destination country as well as the euclidean distances between the countries centers of gravity. The spatial neighbourhood (W) is defined by a decay function ($w_{ij} = exp(-tt_{ij})$) of the car travel times between the capitals of the regions.

$$\begin{aligned}\log trade_{ij} &= \alpha + \rho W \log export_{ij} + \beta_1 \log POP_i + \beta_2 \log POP_j \\ &\quad + \gamma_1 \log GDP_i + \gamma_2 \log GDP_j \\ &\quad + \delta \log dist_{ij} + \epsilon_{ij}.\end{aligned}$$

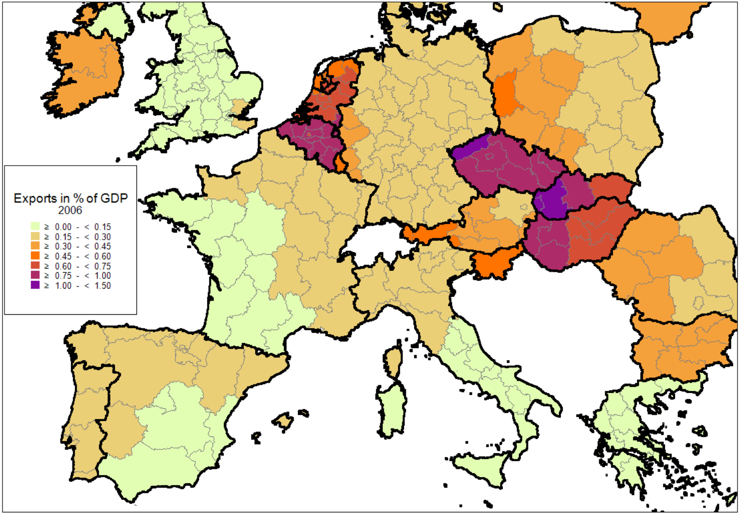
Aggregate Gravitiy Estimation

Table: SAR Chow-Lin Flow Estimation Results

Dependent Variable	Trade Flows		
	Coef	Std. Err.	
c	1.91	0.38	***
GDP origin	0.93	0.04	***
GDP destination	0.59	0.04	***
POP origin	0.00	0.04	
POP destination	0.35	0.05	***
Distance	-1.25	0.05	***
Rho	0.02	0.01	***
R^2		0.87	
Nobs		702	

Predicted NUTS-2 Exports in Percent of GDP

The following figure shows the predicted regional exports per GDP in percent ($0.1 = 10\%$).



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Conclusions

Internal flows are different from external flows if it comes to aggregation!

ML approach is better.

A Bayesian version is possible: Bayes-Chow-Lin for Internal Flows.