Tail dependence models for risk management

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A multivariate stochastic model aims at describing the behavior of several random phenomena that are connected each other. Generally, it can be represented by a d–dimensional random vector $\mathbf{X} = (X_1, \ldots, X_d)$ $(d \geq 2)$ and its associated joint probability distribution function $F_{\mathbf{X}}(\mathbf{x}) = \mathbb{P}(X_1 \leq x_1, \ldots, X_d \leq x_d)$ for every $\mathbf{x} \in \mathbb{R}^d$. In quantitative risk management, a multivariate stochastic model usually aims at calculating suitable risky quantities that are related to the phenomenon under consideration. In Portfolio Management, for instance, one is interested in calculating the Value-at-risk (or any other risk measure) of the aggregate position $X^+ = X_1 + \cdots + X_d$. In Credit Risk, instead, one is more interested in the expected number of defaults (in a given time period) or k–th default time, etc.

Recently, a common approach to deal with such problem consists of two steps. First, the the probabilistic behaviour of each component X_i is investigated; then the relationships among the different vector components are described via a suitable *copula*. Copulas are mathematical objects that fully capture the rank-invariant dependence among random variables and can offer a great flexibility in building multivariate stochastic models. However, while the selection of the probability law for each single component X_i is standard practice, the problem of constructing and/or selecting a suitable copula associated with $\mathbf{X}$ is currently an active research fields. In particular, from a risk management perspective, special care should be devoted to the description of the dependence in the *tails* of the distribution function $F_{\mathbf{X}}$.

Here we focus on some selected investigations about tail dependence (as described by means of copulas) and its possible applications. Our aim is to provide some theoretical, computational and graphical tools that may help the decision maker in the correct identification of linkages among different phenomena especially in a risky scenario. Specifically, we will describe special classes of copulas that can capture different tail dependence behavior and present some statistical inference procedures. Moreover, we provide graphical tools to understand the tail behaviour of a joint probability distribution at a finite scale and discover extreme linkages in a set of different variables.

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