Unique Word Prefix in SC/FDE and OFDM: A Comparison

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***Abstract*—The concept of using unique word (UW) prefixes instead of cyclic prefixes (CPs) is already well investigated for single carrier systems with frequency domain equalization (SC/FDE). In OFDM (orthogonal frequency division multiplexing), where the data symbols are specified in frequency domain, the introduction of UWs - which are specified in time domain - is not that straight forward. In this paper we show how unique words can also be introduced in OFDM symbols. In OFDM our proposed method introduces correlations between subcarrier symbols. This allows to apply a highly efficient LMMSE (linear minimum mean square error) receiver. Throughout this paper we discuss the similarities and differences of UW-SC/FDE and UW-OFDM transmitter/receiver processing, and we present simulation results in indoor multipath environments.**

I. Introduction

Both, OFDM [1] and SC/FDE [2]-[6], use a blockwise data structure with guard intervals in between subsequent blocks. Usually the guard intervals are implemented as cyclic prefixes. The technique of using UWs instead of CPs has already been investigated in-depth for SC/FDE systems, where the introduction of unique words in time domain is straight forward [7]-[9] since the data symbols are also defined in time domain. In this paper we present a concept that allows to introduce UWs in OFDM time domain symbols, even though the data QAM (quadrature amplitude modulation) symbols are defined in frequency domain. Furthermore, receiver processing concepts are discussed for UW-SC/FDE as well as for UW-OFDM.

Fig. 1 compares the transmit data structure of CP- and UW-based transmission in time domain. Both structures make sure that the linear convolution of a block with the impulse response of a dispersive (e.g. multipath) channel appears as a cyclic convolution at the receiver side. Nevertheless, there are also some fundamental differences between CP- and UW-based transmission:

- The UW is part of the DFT (discrete Fourier transform)-interval, whereas the CP is not.
- The CP is random, whereas the UW is a known deterministic sequence. Therefore, the UW can advantageously be utilized for synchronization [10] and channel estimation purposes [9].

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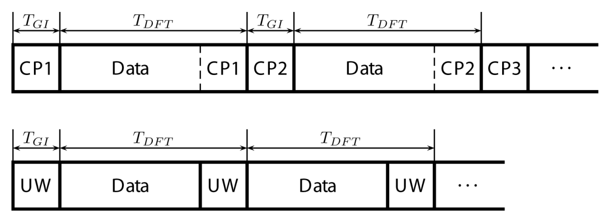

Fig. 1. Transmit data structure using CPs (above) or UWs (below).

Both statements hold for OFDM- as well as for SC/FDE-systems. However, in OFDM our concept of introducing UWs in time domain additionally leads to correlations along the subcarrier symbols. These correlations can advantageously be used as a-priori information at the receiver to significantly improve the BER (bit error ratio) behavior in frequency selective environments.

The rest of the paper is organized as follows: In section II we briefly review the concept of UW-SC/FDE. Furthermore, we discuss a somewhat unusual LMMSE receiver concept. The BER performance of this receiver is compared to the conventional SC/FDE receiver in indoor multipath environments. In section III we describe our approach of how to introduce unique words in OFDM symbols, and we discuss an LMMSE receiver that exploits the correlations introduced at the transmitter side. The novel UW-OFDM concept is compared to classical CP-OFDM by means of simulation results. For this, the IEEE 802.11a WLAN (wireless local area networks) standard serves as reference system.

Notation

Lower-case bold face variables ($\mathbf{a}, \mathbf{b}$,...) indicate vectors, and upper-case bold face variables ($\mathbf{A}, \mathbf{B}$,...) indicate matrices. To distinguish between time and frequency domain variables, we use a tilde to express frequency domain vectors and matrices ($\tilde{\mathbf{a}}, \tilde{\mathbf{A}}$,...), respectively. We further use $\mathbb{C}$ to denote the set of complex numbers, $\mathbf{I}$ to denote the identity matrix, $(\cdot)^T$ to denote transposition, $(\cdot)^H$ to denote conjugate transposition, and $E[\cdot]$ to denote expectation.

II. UW Prefix in SC/FDE

We briefly review the transmit processing and receive data estimation processing in UW based SC/FDE.

A. UW-SC/FDE Transmitter Processing

The introduction of UWs in SC/FDE time domain blocks is well known and straight forward. Let $\mathbf{d} \in \mathbb{C}^{N_d \times 1}$ denote the vector of QAM-symbols, and $\mathbf{u} \in \mathbb{C}^{N_u \times 1}$ be the vector of UW symbols, then a length $N = N_d + N_u$ transmit block is built by $\mathbf{x} = \begin{bmatrix}\mathbf{d}^T & \mathbf{u}^T\end{bmatrix}^T$. The subsequent baseband processing typically consists of up-sampling and pulse shaping.

B. UW-SC/FDE Receiver Processing

1) Diagonal Equalizer Structure: At the receiver side the block based processing can be implemented as follows [10]: A block sampled at twice the symbol rate is transformed to frequency domain with a $2N$-point FFT (fast Fourier transform), matched filtered, down-sampled to symbol rate (typically implemented in frequency domain), and finally a symbol rate frequency domain equalizer is applied. The equalizer input vector can be modeled as

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\mathbf{F}_N\mathbf{x} + \tilde{\mathbf{v}}, \tag{1}$$

where $\tilde{\mathbf{v}}$ is the down-sampled (colored) matched filter output frequency domain noise vector, $\mathbf{F}_N$ is the N-point DFT matrix, and $\tilde{\mathbf{H}}$ is the diagonal channel matrix whose main diagonal elements represent the symbol rate frequency response model of the system consisting of the transmit filter, the multipath channel and the matched filter. Note that this representation corresponds to the Bayesian linear model [11]. In the equalizer derivations it is typically assumed that $\mathbf{x}$ is a random vector consisting of uncorrelated zero mean data symbols with variance σ_d^2. We further assume uncorrelated zero mean time domain noise with variance σ_n^2 at the input of the FFT, and we assume the implementation of a matched filter matched to the channel distorted transmit pulse. With the help of the Bayesian Gauss-Markov theorem [11], the Bayesian LMMSE equalizer for theses assumptions can easily shown to be a diagonal matrix given by

$$\tilde{\mathbf{E}}_1 = \left(\tilde{\mathbf{H}} + \frac{\sigma_n^2}{\sigma_d^2}\mathbf{I}\right)^{-1}. \tag{2}$$

By applying this equalizer to $\tilde{\mathbf{y}}$ and after going back to time domain by an IFFT (inverse FFT), we result at the estimates

$$\begin{bmatrix}\hat{\mathbf{d}} \\ \hat{\mathbf{u}}\end{bmatrix} = \mathbf{F}_N^{-1}\tilde{\mathbf{E}}_1\tilde{\mathbf{y}}. \tag{3}$$

Note that the UW symbols are treated like the random data symbols in this approach. They are equalized like data symbols which is quite useful for some parameter estimation and synchronization tasks.

2) Non-Diagonal Equalizer Structure: For completeness and because of comparison reasons with UW-OFDM receiver processing we now introduce a somewhat untypical UW-SC/FDE receiver, namely the ”true” Bayesian LMMSE equalizer. This one is obtained by assuming $\mathbf{u}$ as deterministic instead of random. This can be incorporated in the model for the received block by partitioning $\mathbf{F}_N$ as $\mathbf{F}_N = \begin{bmatrix}\mathbf{M_1} & \mathbf{M_2}\end{bmatrix}$ with $\mathbf{M}_1 \in \mathbb{C}^{N \times N_d}$ and $\mathbf{M}_2 \in \mathbb{C}^{N \times N_u}$. Hence, a received block can be modelled as $\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\mathbf{M}_1\mathbf{d} + \tilde{\mathbf{H}}\mathbf{M}_2\mathbf{u} + \tilde{\mathbf{v}}$. With the frequency domain version $\tilde{\mathbf{u}} = \mathbf{M}_2\mathbf{u} = \mathbf{F}_N\begin{bmatrix}\mathbf{0}^T & \mathbf{u}^T\end{bmatrix}^T$ of the UW this can be re-written as $\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\mathbf{M}_1\mathbf{d} + \tilde{\mathbf{H}}\tilde{\mathbf{u}} + \tilde{\mathbf{v}}$. Note that (assuming that the channel matrix $\tilde{\mathbf{H}}$ or at least an estimate of the same is available) $\tilde{\mathbf{H}}\tilde{\mathbf{u}}$ represents a known signal contained in the received vector $\tilde{\mathbf{y}}$, and the LMMSE equalization procedure can now easily be shown to be

$$\hat{\mathbf{d}} = \tilde{\mathbf{E}}_2\left(\tilde{\mathbf{y}} - \tilde{\mathbf{H}}\tilde{\mathbf{u}}\right) \tag{4}$$

with

$$\tilde{\mathbf{E}}_2 = \mathbf{M}_1^H\left(\tilde{\mathbf{H}}\mathbf{M}_1\mathbf{M}_1^H + \frac{N\sigma_n^2}{\sigma_d^2}\mathbf{I}\right)^{-1}. \tag{5}$$

Note that different to $\tilde{\mathbf{E}}_1$ the equalizer $\tilde{\mathbf{E}}_2$ additionally performs the transform back to time domain, therefore no subsequent IFFT procedure is required. However, the equalizer can no longer be described by a diagonal matrix which definitely leads to a significantly higher complexity for the equalizer determination (5) as well as for the equalization procedure (4). Note further that the UW symbols are not equalized by this procedure. In the next section we will show that our proposed UW-OFDM receiver shows huge similarity to this approach.

C. Simulation results

Table I depicts the main PHY-parameters of the investigated UW-SC/FDE system. The parameters have been adapted to the IEEE 802.11a WLAN standard [12], such that the UW-SC/FDE system achieves the same data rates. In the simulation results shown below only the QPSK mode is applied.

TABLE I
Main PHY parameters of the investigated UW-SC/FDE system.

Modulation schemes	BPSK, QPSK, 16QAM, 64QAM
Coding rates	1/2, 2/3, 3/4
Total number of symbols per block	64
Number of data symbols per block	52
Number of UW symbols per block	12
Block duration	4.33 μs
UW duration (guard time)	812.5 ns
DFT interval	4.33 μs
Duration of one individual symbol	68 ns
Pulse shaping	RRC (α = 0.25)

The same convolutional coder with the industry standard rate 1/2, constraint length 7 code with generator polynomials (133,171) as in [12] has been used. For decoding we applied a soft decision Viterbi algorithm. The multipath channel has been modeled as a tapped delay line, each tap with uniformly distributed phase and Rayleigh distributed magnitude, and with power decaying exponentially. A detailed description of the model can be found in [9]. Fig. 2 shows two typical channel snapshots, both featuring an rms delay spread of 100ns. The frequency response of channel A features two spectral notches within the system's bandwidth, whereas channel B shows no deep fading holes.

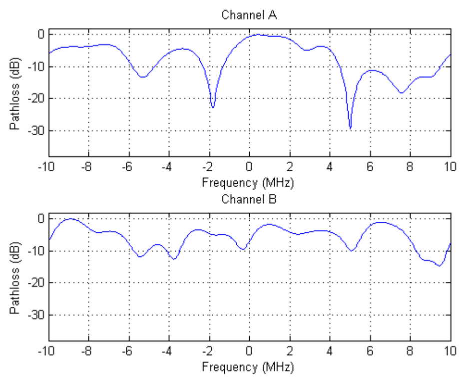

Fig. 2. Frequency domain representation of the indoor multipath channel snapshots A and B.

Fig. 3 and 4 show BER simulation results in the QPSK mode for the channel snapshots shown above for the uncoded case and for the coding rates $r = 3/4$ and $r = 1/2$, respectively. Perfect channel knowledge is assumed in all presented simulation results. It can be seen that the "true" LMMSE equalizer $\tilde{\mathbf{E}}_2$ outperforms the conventional equalizer $\tilde{\mathbf{E}}_1$ for both channels. For channel A the gain is around 0.5dB at a BER of 10^{-6} for the uncoded case, it reduces to around 0.3dB and 0.1dB for $r = 3/4$ and $r = 1/2$, respectively. For channel B the gain is around 0.2dB for the uncoded case, and around 0.15dB and 0.1dB for $r = 3/4$ and $r = 1/2$, respectively. We will refer back to these results in the following section.

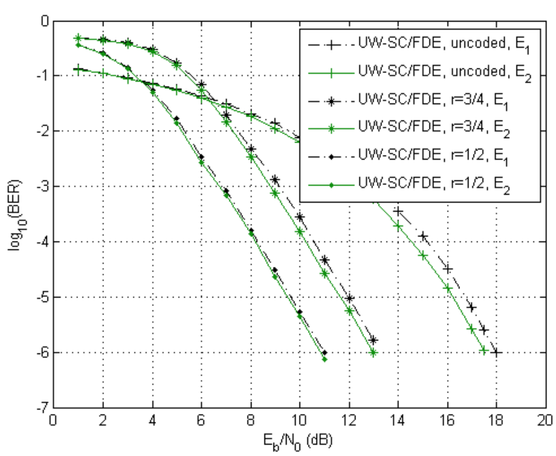

Fig. 3. UW-SC/FDE BER simulation results for two different equalizer structures (QPSK mode, channel A).

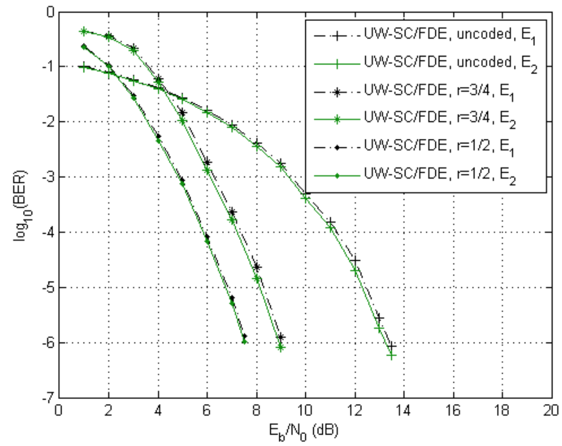

Fig. 4. UW-SC/FDE BER simulation results for two different equalizer structures (QPSK mode, channel B).

III. UW PREFIX IN OFDM

A. UW-OFDM Transmitter Processing

In OFDM the data vector $\tilde{\mathbf{d}} \in \mathbb{C}^{N_d \times 1}$ is defined in frequency domain. Typically, zero subcarriers are inserted at the band edges and at the DC-subcarrier position, which can mathematically be described by a matrix operation $\tilde{\mathbf{x}} = \mathbf{B}\tilde{\mathbf{d}}$ with $\tilde{\mathbf{x}} \in \mathbb{C}^{N \times 1}$ and $\mathbf{B} \in \mathbb{C}^{N \times N_d}$. $\mathbf{B}$ consists of zero-rows at the positions of the zero subcarriers, and of appropriate unit row vectors at the positions of data subcarriers. The vector $\tilde{\mathbf{x}}$ denotes the OFDM symbol in frequency domain. The vector of time domain samples $\mathbf{x} \in \mathbb{C}^{N \times 1}$ is calculated via an IFFT operation $\mathbf{x} = \mathbf{F}_N^{-1}\tilde{\mathbf{x}}$.

We now modify this conventional approach by introducing a pre-defined UW sequence $\mathbf{x}_u$ with $\mathbf{x}_u \in \mathbb{C}^{N_u \times 1}$, which shall form the tail of the time domain vector (as in UW-SC/FDE), which we now denote by $\mathbf{x}'$. Hence, $\mathbf{x}'$ consists of two parts and is given by $\mathbf{x}' = \begin{bmatrix} \mathbf{x}_d^T & \mathbf{x}_u^T \end{bmatrix}^T$ with $\mathbf{x}_d \in \mathbb{C}^{(N-N_u) \times 1}$, whereas only $\mathbf{x}_d$ is random and affected by the data. We use a two-step approach for the generation of the so-defined vector $\mathbf{x}'$ ([13], [14]):

- Generate a zero UW $\mathbf{x} = \begin{bmatrix} \mathbf{x}_d^T & \mathbf{0}^T \end{bmatrix}^T$ such that $\mathbf{x} = \mathbf{F}_N^{-1}\tilde{\mathbf{x}}$.
- Add the UW in time domain such that $\mathbf{x}' = \mathbf{x} + \begin{bmatrix} \mathbf{0}^T & \mathbf{x}_u^T \end{bmatrix}^T$.

We now describe the first step in detail: As in conventional OFDM, the QAM data symbols and the zero subcarriers are specified in frequency domain in vector $\tilde{\mathbf{x}}$, but here in addition the zero-word is specified in time domain as part of the vector $\mathbf{x}$. As a consequence, the linear system of equations $\mathbf{x} = \mathbf{F}_N^{-1}\tilde{\mathbf{x}}$ can only be fulfilled by reducing the number N_d of data subcarriers, and by introducing a set of redundant subcarriers instead. We let the redundant subcarriers form the vector $\tilde{\mathbf{r}} \in \mathbb{C}^{N_r \times 1}$ with $N_r = N_u$, further introduce a permutation matrix $\mathbf{P} \in \mathbb{C}^{(N_d+N_r) \times (N_d+N_r)}$, and form an

OFDM symbol (containing $N-N_d-N_r$ zero-subcarriers) in frequency domain by

$$\tilde{\mathbf{x}} = \mathbf{B}\mathbf{P}\begin{bmatrix}\tilde{\mathbf{d}}\\ \tilde{\mathbf{r}}\end{bmatrix}. \quad (6)$$

We will detail the reason for the introduction of the permutation matrix and its specific construction shortly below. $\mathbf{B} \in \mathbb{C}^{N\times(N_d+N_r)}$ is again a trivial matrix that inserts the usual zero subcarriers. Fig. 5 illustrates this approach in a graphical way. Note that our approach of introducing the zero word as guard interval is fundamentally different to zero padded (ZP)-OFDM [15]. In UW-OFDM the zero word is part of the DFT interval, whereas this is not the case in ZP-OFDM.

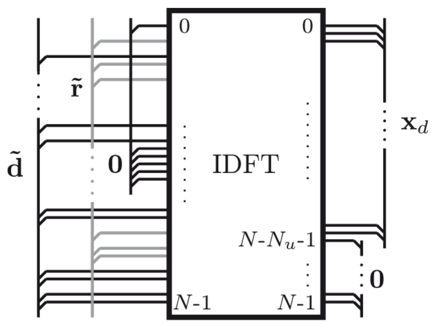

Fig. 5. Time- and frequency-domain view of an OFDM symbol in UW-OFDM.

The time - frequency relation of the OFDM symbol can now be written as

$$\mathbf{F}_N^{-1}\mathbf{B}\mathbf{P}\begin{bmatrix}\tilde{\mathbf{d}}\\ \tilde{\mathbf{r}}\end{bmatrix} = \begin{bmatrix}\mathbf{x}_d\\ \mathbf{0}\end{bmatrix}. \quad (7)$$

With $\mathbf{M} = \mathbf{F}_N^{-1}\mathbf{B}\mathbf{P} = \begin{bmatrix}\mathbf{M}_{11} & \mathbf{M}_{12}\\ \mathbf{M}_{21} & \mathbf{M}_{22}\end{bmatrix}$, where $\mathbf{M}_{ij}$ are appropriate sized sub-matrices, it follows that $\mathbf{M}_{21}\tilde{\mathbf{d}} + \mathbf{M}_{22}\tilde{\mathbf{r}} = \mathbf{0}$, and hence $\tilde{\mathbf{r}} = -\mathbf{M}_{22}^{-1}\mathbf{M}_{21}\tilde{\mathbf{d}}$. With the matrix

$$\mathbf{T} = -\mathbf{M}_{22}^{-1}\mathbf{M}_{21} \quad (8)$$

($\mathbf{T} \in \mathbb{C}^{N_r\times N_d}$), the vector of redundant subcarriers can thus be determined by the linear mapping

$$\tilde{\mathbf{r}} = \mathbf{T}\tilde{\mathbf{d}}. \quad (9)$$

$\mathbf{T}$ introduces correlation in the vector $\tilde{\mathbf{x}}$ of frequency domain samples of an OFDM symbol.

We notice that the construction of $\mathbf{T}$ and therefore also the variances of the redundant subcarrier symbols highly depend on the positions of the redundant subcarriers within the frequency domain vector $\tilde{\mathbf{x}}$. It turns out that the energy of the redundant subcarrier symbols almost explodes without the use of the permutation matrix. The problem can be solved by an optimized permutation of the data and redundant subcarrier symbols. In [13] we suggested to select $\mathbf{P}$ such that $\mathrm{trace}\left(\mathbf{T}\mathbf{T}^H\right)$ becomes minimum. It can be shown (see again [13]) that this provides minimum energy on the redundant subcarrier symbols on average (when averaging over all possible data vectors $\tilde{\mathbf{d}}$).

In the style of block coding theory we use the notation

$$\tilde{\mathbf{c}} = \mathbf{P}\begin{bmatrix}\tilde{\mathbf{d}}\\ \tilde{\mathbf{r}}\end{bmatrix} = \mathbf{P}\begin{bmatrix}\mathbf{I}\\ \mathbf{T}\end{bmatrix}\tilde{\mathbf{d}} = \mathbf{G}\tilde{\mathbf{d}}, \quad (10)$$

($\tilde{\mathbf{c}} \in \mathbb{C}^{(N_d+N_r)\times 1}$, $\mathbf{G} \in \mathbb{C}^{(N_d+N_r)\times N_d}$) for the non-zero part of $\tilde{\mathbf{x}}$, such that $\tilde{\mathbf{x}} = \mathbf{B}\tilde{\mathbf{c}}$. $\mathbf{G}$ and $\tilde{\mathbf{c}}$ can be interpreted as a code generator matrix and a complex valued codeword, respectively.

In the second step the transmit symbol $\mathbf{x}'$ is generated by adding the unique word: $\mathbf{x}' = \mathbf{x} + \begin{bmatrix}\mathbf{0}^T & \mathbf{x}_u^T\end{bmatrix}^T$. The frequency domain version $\tilde{\mathbf{x}}_u \in \mathbb{C}^{N\times 1}$ of the UW is defined by $\tilde{\mathbf{x}}_u = \mathbf{F}_N\begin{bmatrix}\mathbf{0}^T & \mathbf{x}_u^T\end{bmatrix}^T$. Note that $\mathbf{x}'$ can also be written as $\mathbf{x}' = \mathbf{F}_N^{-1}(\tilde{\mathbf{x}} + \tilde{\mathbf{x}}_u) = \mathbf{F}_N^{-1}(\mathbf{B}\mathbf{G}\tilde{\mathbf{d}} + \tilde{\mathbf{x}}_u)$.

B. UW-OFDM Receiver Processing

After the transmission over a multipath channel and after the common FFT operation, the non-zero part $\tilde{\mathbf{y}} \in \mathbb{C}^{(N_d+N_r)\times 1}$ of a received OFDM frequency domain symbol can be modeled as

$$\tilde{\mathbf{y}} = \mathbf{B}^T\mathbf{F}_N\mathbf{H}\mathbf{F}_N^{-1}(\mathbf{B}\mathbf{G}\tilde{\mathbf{d}} + \tilde{\mathbf{x}}_u) + \mathbf{B}^T\mathbf{F}_N\mathbf{n}, \quad (11)$$

where $\mathbf{H}$ denotes a cyclic convolution matrix ($\mathbf{H} \in \mathbb{C}^{N\times N}$) constructed of the channel impulse response coefficients, and $\mathbf{n} \in \mathbb{C}^{N\times 1}$ represents a noise vector with the covariance matrix $\sigma_n^2\mathbf{I}$. The multiplication with $\mathbf{B}^T$ excludes the zero subcarriers from further operation. The matrix $\mathbf{F}_N\mathbf{H}\mathbf{F}_N^{-1}$ is diagonal and contains the sampled channel frequency response on its main diagonal. $\tilde{\mathbf{H}} = \mathbf{B}^T\mathbf{F}_N\mathbf{H}\mathbf{F}_N^{-1}\mathbf{B}$ with $\tilde{\mathbf{H}} \in \mathbb{C}^{(N_d+N_r)\times(N_d+N_r)}$ is a down-sized version of the latter excluding the entries corresponding to the zero subcarriers. (Note that we use the same symbol $\tilde{\mathbf{H}}$ in the system models of UW-OFDM and UW-SC/FDE, although the channel matrices in the two system approaches are defined in a different way.) The received symbol can now be written as

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\mathbf{G}\tilde{\mathbf{d}} + \tilde{\mathbf{H}}\mathbf{B}^T\tilde{\mathbf{x}}_u + \tilde{\mathbf{v}} \quad (12)$$

with the noise vector $\tilde{\mathbf{v}} = \mathbf{B}^T\mathbf{F}_N\mathbf{n}$. Note that (assuming that the channel matrix $\tilde{\mathbf{H}}$ or at least an estimate of the same is available) $\tilde{\mathbf{H}}\mathbf{B}^T\tilde{\mathbf{x}}_u$ represents a known signal contained in the received vector $\tilde{\mathbf{y}}$. In order to determine the Bayesian LMMSE estimator, let $\tilde{\mathbf{y}}' = \tilde{\mathbf{y}} - \tilde{\mathbf{H}}\mathbf{B}^T\tilde{\mathbf{x}}_u$ such that $\tilde{\mathbf{y}}' = \tilde{\mathbf{H}}\mathbf{G}\tilde{\mathbf{d}} + \tilde{\mathbf{v}}$. Again applying the Bayesian Gauss-Markov theorem, the LMMSE estimator follows to

$$\hat{\tilde{\mathbf{d}}} = \mathbf{C}_{\tilde{d}\tilde{d}}\mathbf{G}^H\tilde{\mathbf{H}}^H(\tilde{\mathbf{H}}\mathbf{G}\mathbf{C}_{\tilde{d}\tilde{d}}\mathbf{G}^H\tilde{\mathbf{H}}^H + \mathbf{C}_{\tilde{v}\tilde{v}})^{-1}\tilde{\mathbf{y}}'. \quad (13)$$

With $\mathbf{C}_{\tilde{d}\tilde{d}} = \sigma_d^2\mathbf{I}$ and $\mathbf{C}_{\tilde{v}\tilde{v}} = E\left[\tilde{\mathbf{v}}\tilde{\mathbf{v}}^H\right] = N\sigma_n^2\mathbf{I}$ we immediately obtain

$$\hat{\tilde{\mathbf{d}}} = \mathbf{G}^H\left(\mathbf{G}\mathbf{G}^H + \frac{N\sigma_n^2}{\sigma_d^2}(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}})^{-1}\right)^{-1}\tilde{\mathbf{H}}^{-1}\tilde{\mathbf{y}}'. \quad (14)$$

With the Wiener smoothing matrix

$$\tilde{\mathbf{W}} = \mathbf{G}^H\left(\mathbf{G}\mathbf{G}^H + \frac{N\sigma_n^2}{\sigma_d^2}(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}})^{-1}\right)^{-1}, \quad (15)$$

the LMMSE estimator can now compactly be written as

$$\hat{\tilde{\mathbf{d}}} = \tilde{\mathbf{W}}\tilde{\mathbf{H}}^{-1}(\tilde{\mathbf{y}} - \tilde{\mathbf{H}}\mathbf{B}^T\tilde{\mathbf{x}}_u). \quad (16)$$

From (16) we can conclude that the LMMSE receiver consists of a zero forcing stage (as it also appears in conventional CP-OFDM) which is followed by a Wiener smoothing operation for noise reduction on the subcarriers. The Wiener smoother exploits the correlations between subcarrier symbols which have been introduced by (9) at the transmitter. The equalization process shows great similarities to the equalization steps (4) and (5) described for the UW-SC/FDE system. Of course the zero forcing and the smoothing operation can be implemented in one combined single matrix multiplication operation. Furthermore, the UW elimination could also be performed by subtracting $\mathbf{B}^T\tilde{\mathbf{x}}_u$ after the ZF-stage.

We notice that the error $\tilde{\mathbf{e}} = \tilde{\mathbf{d}} - \hat{\tilde{\mathbf{d}}}$ has zero mean, and its covariance matrix is given by [11]

$$\mathbf{C}_{\tilde{e}\tilde{e}} = \sigma_d^2 \left(\mathbf{I} - \tilde{\mathbf{W}}\mathbf{G}\right). \quad (17)$$

In our system simulations the main diagonal of matrix $\mathbf{C}_{\tilde{e}\tilde{e}}$ is used in a soft decision Viterbi decoder to specify the varying noise variances along the data symbols after equalization and Wiener smoothing.

C. Simulation Results

We compare our novel UW-OFDM approach with the classical CP-OFDM concept. The IEEE 802.11a WLAN standard [12] serves as reference system. We apply the same parameters for UW-OFDM as in [12] wherever possible, the most important parameters are specified in Table II.

TABLE II
MAIN PHY PARAMETERS OF THE INVESTIGATED UW-OFDM SYSTEM.

	802.11a	UW-OFDM
Modulation schemes	BPSK, QPSK, 16QAM, 64QAM	BPSK, QPSK, 16QAM, 64QAM
Coding rates	1/2, 2/3, 3/4	1/2, 2/3, 3/4
Used subcarriers per block	52	52
Data subcarriers	48	36
Additional subcarriers	4 (pilots)	16 (redundant)
DFT period	3.2 μs	3.2 μs
Guard duration	800 ns (CP)	800 ns (UW)
Total OFDM symbol duration	4 μs	3.2 μs

The indices of the redundant subcarriers are chosen to be {2, 6, 10, 14, 17, 21, 24, 26, 38, 40, 43, 47, 50, 54, 58, 62}. This choice, which can easily also be described by (6) with an appropriately constructed matrix $\mathbf{P}$, minimizes the total energy of the redundant subcarriers on average [13]. Note that in conventional CP-OFDM like in the WLAN standard, the total length of an OFDM symbol is given by $T_{GI} + T_{DFT}$. However, the guard interval is part of the DFT period in our UW-OFDM approach. Therefore, both systems show almost identical bandwidth efficiency. In our approach

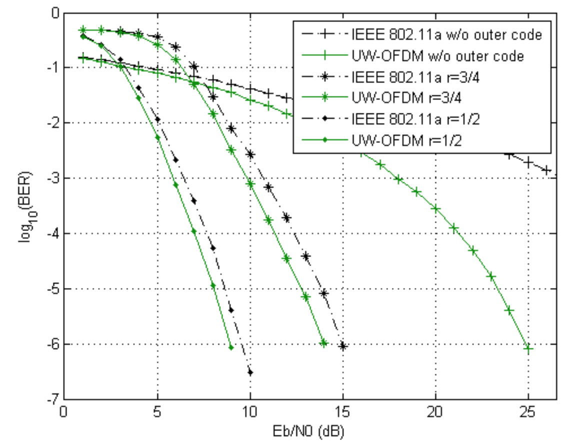

Fig. 6. BER comparison between the novel UW-OFDM approach and the IEEE 802.11a standard (QPSK mode, channel A).

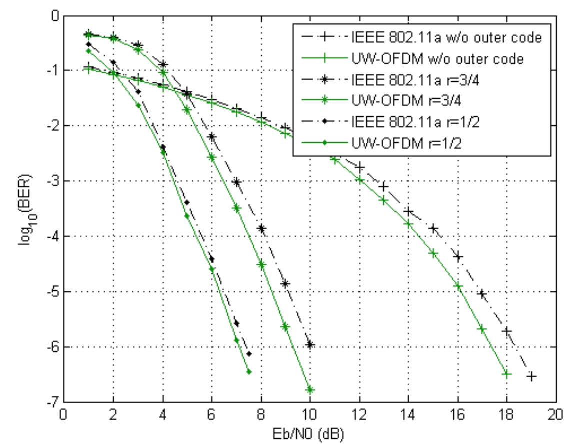

Fig. 7. BER comparison between the novel UW-OFDM approach and the IEEE 802.11a standard (QPSK mode, channel B).

the UW shall take over the synchronization tasks which are normally performed with the help of the 4 pilot subcarriers. In order to make a fair BER over E_b/N_0 comparison, the energy of the UW related to the total energy of a transmit symbol is set to 4/52, which exactly corresponds to the total energy of the 4 pilots related to the total energy of a transmit symbol in the IEEE standard.

In Fig. 6 the BER-behavior of the novel UW-OFDM approach and the IEEE 802.11a standard are compared, both in QPSK-mode and for the highly frequency selective channel A displayed in Fig. 2. For both systems the same convolutional coder as described in [12] is used. Note that due to the different number of data QAM symbols per OFDM symbol in our UW-OFDM system setup compared to IEEE 802.11a, the 802.11a interleaver had to be slightly adapted for the UW-OFDM system. Again perfect channel knowledge is assumed in all presented simulation results. In the case no further outer code is used, UW-OFDM significantly outperforms CP-

OFDM. This can be explained by the significant noise reduction on heavily attenuated subcarriers achieved by the Wiener smoother [13]. For the coding rates $r = 3/4$ and $r = 1/2$ the novel UW-OFDM approach still achieves a gain of 0.9dB and 0.6dB at a bit error ratio of 10^{-6}, respectively. Fig. 7 shows the results for channel B, cf. Fig. 2. Even though the gains are reduced, we notice similar tendencies. For $r = 1$ the performance gain of UW-OFDM is 0.9dB at a bit error ratio of 10^{-6}, and this turns to 0.7dB and to 0.3dB for $r = 3/4$ and $r = 1/2$, respectively.

We are aware that comparing different systems in a fair way is always difficult. Nevertheless, we ask the reader to compare the corresponding BER simulation curves for the UW-SC/FDE and the UW-OFDM system. The bandwidth efficiencies are almost identical for the chosen parameter setups, also all other parameters are very comparable, and finally we used the same channel snapshots (which had to be resampled for the UW-OFDM system) for our presented simulation results. We identify the typical trend, namely that uncoded UW-SC/FDE outperforms uncoded OFDM (also UW-OFDM which shows much better performance than CP-OFDM), while the OFDM performance improves significantly by using low coding rates.

IV. Conclusion

In this work we compared the transmitter and receiver processing for UW-SC/FDE and UW-OFDM. The introduction of UWs in SC/FDE is well known and straight forward, while the introduction of UWs in OFDM is much more sophisticated. Our approach of introducing UWs in OFDM causes correlations between subcarrier symbols. Therefore UW-OFDM allows for LMMSE equalization similar as in UW-SC/FDE receivers, and it clearly outperforms classical CP-OFDM in typical frequency selective indoor scenarios.

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