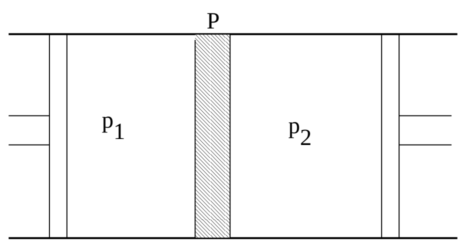
13. (a) For the m-dimensional distribution $f(\mathbf{x})$ defined in problem 3.(a), calculate the average $\bar{E}$ of $E = |\mathbf{x}|^2$.

(b) Calculate the variance ΔE of E. ΔE is defined as $\Delta E = \sqrt{\bar{E^2} - (\bar{E})^2}$. How does the ratio $\Delta E/\bar{E}$ depend on the dimension m?

14. Joule-Thomson effect: A gas of temperature T under pressure p_1 is released adiabatically and slowly (e.g. through a porous plug P) into a chamber held at pressure p_2. Show that in this adiabatic process the enthalpy is constant. Prove the following relation (the Joule-Thomson coefficient) for the change of temperature:

$$\left(\frac{dT}{dp}\right)_{\text{J.-Th.}} = \frac{V}{C_p}\left[T\left(\frac{dV}{dT}\right)_p - V\right]$$

Calculate and discuss the Joule-Thomson coefficient for the ideal gas and the Van der Waals gas. The curve $(\frac{dT}{dp})_{\text{J.-Th.}} = 0$ is called inversion curve and defines the inversion pressure p_{inv}. Calculate p_{inv} for the Van der Waals gas.



15. A particle moves along a one-dimensional lattice, jumping one lattice point to the right with probability p and one lattice point to the left with probability $1-p$.

(a) Show that after N jumps, a particle that starts at point 0 is at lattice point x with a probability given by the binomial distribution

$$P(n, N; p) = p^n(1-p)^{N-n}\binom{N}{n}, \quad \text{where } n = \frac{x+N}{2}$$

(b) What is the average position $\bar{x}$ of the particle after N jumps?

(c) Show that for large N, the binomial distribtion becomes the Gaussian distribution. For simplicity, you can assume $p = \frac{1}{2}$. [Hint: use Stirling's formula $\ln M! \approx M\ln M - M$ for large M.]

16. Let p_i, $i = 1, \ldots, N$, be the probability to find a system in state i (for example, for a dice we would have $N = 6$ and $p_i = \frac{1}{6}$). Note that $\sum_{i=1}^N p_i = 1$. We define an "uncertainty function" $H(p_1, \ldots, p_N) = -\sum_{i=1}^N p_i \ln p_i$.

(a) Show that the uncertainty H vanishes if $p_i = 1$ for a single state i and $p_j = 0$ for $j \neq i$. Show that H has a maximum if all p_i are the same.

(b) Generalize the uncertainty function H to the case of a continuous probability distribution $p(x)$, and calculate H for the Maxwell-Boltzmann distribution.