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Statistical Methods

Tutorial in WS 2016/17 for Monday, 30.01.17
from 12:00-13:30 in MT 327

Tutorial 7

p-value, confidence intervals and interval estimation

Exercise 37 (p-value)
Let $X_1, \ldots, X_n$ be random from $N(\mu, \sigma^2)$ with unknown μ and σ. Consider the hypothesis test: $H_0 : \mu \leq \mu_0$ versus $H_1 : \mu > \mu_0$. Derive the LRT for these hypothesis and show that it rejects H_0 for large values of

$$\frac{\bar{X} - \mu_0}{S/\sqrt{n}}.$$

Calculate the p-value for this test.

Exercise 38 (Pivotal quantities)
Prove the Theorem on slide 7 of week 12.

Exercise 39 (Confidence intervals 1)
Let $X_1, \ldots, X_n$ be iid random variables with pdf

$$f(x; \mu) = e^{-(x-\mu)} \mathbb{1}_{[\mu,\infty)}(x).$$

a) Find a sufficient statistic T for μ and calculate its pdf f_T.

b) Calculate a $1 - \alpha$ confidence interval for μ inverting the LRT.

Exercise 40 (Confidence intervals 2)
Continuation of Exercise 39.

c) Calculate a $1 - \alpha$ confidence interval for μ using the pivotal method.

d) Which is the shortest confidence interval between those in b) and c)?

Exercise 41 (Coverage probability vs. credible probability)
Let $X \sim N(\mu, 1)$ and consider the confidence interval

$$C_a(x) = \{\mu : \min\{0, x-a\} \leq \mu \leq \max\{0, x+a\}\}.$$

a) For $a = 1.645$, prove that the coverage probability of $C_a(x)$ is exactly 0.95 for all μ, with the exception of $\mu = 0$, where the coverage probability is 1.

b) Consider the noninformative prior $\pi(\mu) = 1$. Using this prior and taking $a = 1.645$, show that the posterior credible probability of $C_a(x)$ is exactly 0.90 for $-1.645 \leq x \leq 1.645$ and increases to 0.95 as $|x| \to \infty$.

Exercise 42 (Confidence interval of minimal length)
Let $X_1, \ldots, X_n$ be independent with pdf

$$f_{X_i}(x; \theta) = e^{i\theta - x} \mathbb{1}_{[i\theta, \infty)}(x).$$

a) Prove that

$$T = \min_{i \in \{1, \ldots, n\}} \frac{X_i}{i}$$

is a sufficient statistic for θ and determine its pdf $f_T(t)$.

b) Based on T, find the $1-\alpha$ confidence interval for θ of the form $[T+a, T+b]$ which is of minimal length.