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Statistical Methods

Tutorial in WS 2017/18 for Monday, 16.10.17
from 12:00-13:30 in S2 053

Tutorial 1

Revision: Probability theory

Exercise 1 (Conditional probability)
Let A, B, C events such that $\mathbb{P}(A)\mathbb{P}(B)\mathbb{P}(C) > 0$.

1. Determine $\mathbb{P}(A|B)$ if (a) $A \cap B = \emptyset$, (b) $A \subset B$ and (c) $B \subset A$.
2. Let $\mathbb{P}(A|B) > \mathbb{P}(A)$. Show that $\mathbb{P}(B|A) > \mathbb{P}(B)$.
3. Show that $\mathbb{P}(A \cap B \cap C) = \mathbb{P}(A|B \cap C)\mathbb{P}(B|C)\mathbb{P}(C)$. Use that to compute $\mathbb{P}(A \cap B|C)$.

Exercise 2 (Applications for conditional probability)

1. Two fair dice are thrown behind your back and you receive the information that the total sum of the two dice is not greater that 3. What is the probability that the two dice have the same result with and without the given information?
2. Two companies deliver cucumbers to a trade chain. The Austrian company produces 1000 cucumbers from which 100 are curved. The Italian company produces 2000 cucumbers from which 150 are curved. You buy a curved cucumber in the store. What is the probability that the purchased cucumber originates from Austria?

Exercise 3 (Stochastic independence)

1. Two fair dice are thrown. Let A denote the event that the total sum of all dice is 7. Let B denote the event that the first dice equals 4, let C be the event that the second dice equals 3 and denote $D = B \cap C$. Are A and B independent? Are A and C independent? Are A and D independent?

2. One fair dice is consecutively thrown twice. Examine the events

$$\begin{aligned} A &= \{\text{1st throw is odd}\}, \\ B &= \{\text{2nd throw is odd}\} \text{ and} \\ C &= \{\text{Total sum of throws is odd}\} \end{aligned}$$

w.r.t. to their (pairwise) independence as well as their mutual independence.

Exercise 4 (Simpsons's paradox)
Discuss Simpsons's paradox on slide 14 of week 1 for the two doctors planning to try out a new treatment on patients. Which doctor would you trust?

Exercise 5 (Probability density and distribution functions)
Let $x \in \mathbb{R}$ and k be constant. The probability density function of a *Laplace* distributed random variable X is

$$f_X(x) := k\mathrm{e}^{-\lambda|x|}$$

for $\lambda > 0$. The probability density function of a *Cauchy* distributed random variable Y is

$$f_Y(x) := \frac{k}{a^2 + x^2}$$

for $a > 0$. Work out the following tasks for **either** X **or** Y (or even both).

1. Find the constant k.
2. Determine the distribution function F_X or F_Y.
3. Compute the mean and the variance of X or Y.

Exercise 6 (Moments and moment generating function of random variables)

1. The sample space of Y is $\{-2, -1, 0, 3\}$ and each outcome has probability $1/4$, e.g. $\mathbb{P}(Y = -2) = 1/4$. Calculate $\mathbb{E}[Y], \mathbb{E}[Y^2], \mathrm{Var}(Y)$.
2. Let X be a random variable and $a, b \in \mathbb{R}$. Show that

   $$\mathrm{Var}(aX + b) = a^2\mathrm{Var}(X).$$

3. Denote $M_X(t)$ be the moment generating function of X. Prove that $M_{aX+b}(t) = e^{bt}M_X(at)$ for any $a, b \in \mathbb{R}$.