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Statistical Methods

Tutorial in WS 2017/18 for Monday, 13.11.17
from 12:00-13:30 in S2 053

Tutorial 3

Minimal sufficient and ancillary statistics, moment and maximum likelihood estimators

Exercise 13 (Sufficient statistics)
Let $X_1, \ldots, X_n$ be iid random variables from $Bernoulli(p)$ with unknown $p \in [0,1]$. Is

$$T(\mathbf{X}) = \sum_{i=1}^{n} X_i$$

a sufficient statistic for p?

Exercise 14 (Minimal sufficient statistics 1)
Let $X_1, \ldots, X_n$ be iid random variables from $Beta(\alpha, 1)$ with unknown $\alpha > 0$. Determine a minimal sufficient statistic for α.

Hint: Remember that $\Gamma(t+1) = t\Gamma(t)$ for all $t > 0$.

Exercise 15 (Minimal sufficient statistics 2)
Let $\theta > 0$ and $X_1, \ldots, X_n$ be iid random variables with probability density function

$$f(x;\theta) = \frac{1}{2\theta} \exp\left(-\frac{|x|}{\theta}\right)$$

for $x \in \mathbb{R}$. Perform the following tasks without the results for exponential families.

a) Show that

$$T(\mathbf{X}) = \sum_{i=1}^{n} |X_i|$$

is a sufficient statistic for θ.

b) Find a minimal sufficient statistic for θ.

c) Compute the distribution of $|X_i|, T$ and that of $\mathbf{X}$ given $T = t$.

Exercise 16 (Sufficient and ancillary statistics)

1. Let $X_1, \ldots, X_n$ be an iid sample from a continuous $U[a, b]$ with $a < b$. The unknown parameter is $\boldsymbol{\theta} = (a, b)$. Show that

   $$T(\mathbf{X}) = (X_{(1)}, X_{(n)})$$

   is a sufficient statistic for $\boldsymbol{\theta}$, where $X_{(1)}$ and $X_{(n)}$ are the 1st and nth order statistics, respectively.

2. Let X_1 and X_2 be iid random variables from the discrete distribution satisfying

   $$\mathbb{P}(X = \theta) = \mathbb{P}(X = \theta + 1) = \mathbb{P}(X = \theta + 2) = \frac{1}{3}$$

   for $\theta \in \mathbb{N}$. Consider $R = X_{(2)} - X_{(1)}$ and $M = (X_{(1)} + X_{(2)})/2$, where $X_{(i)}$ is the ith order statistic.

   a) Show that (R, M) is a sufficient statistic for θ.
   b) Show that R is an ancillary statistic for θ.
   c) Show that R and (R, M) are not independent.

Exercise 17 (Fisher information 1)

Prove Theorem 1) on slide 12 of week 5.

Exercise 18 (Fisher information 2)

Prove Theorem 3) on slide 12 of week 5.