

## Statistical Methods

Tutorial in WS 2017/18 for Monday, 08.01.18  
from 12:00-13:30 in S2 053

### Tutorial 6

*Hypothesis testing: Finding and evaluating tests*

#### Exercise 31 (Likelihood ratio test 1)

Let  $X_1, \dots, X_n$  be a random sample from  $N(\mu_X, \sigma_X^2)$  and let  $Y_1, \dots, Y_m$  be an independent random sample from  $N(\mu_Y, \sigma_Y^2)$ . Derive the LRT for  $H_0 : \mu_X = \mu_Y$  versus  $H_1 : \mu_X \neq \mu_Y$  with the assumption that  $\sigma_X^2 = \sigma_Y^2 = \sigma^2$ .

#### Exercise 32 (Likelihood ratio test 2)

Let  $X_1, \dots, X_n$  be a random sample from  $N(0, \sigma_X^2)$  and let  $Y_1, \dots, Y_m$  be an independent random sample from  $N(0, \sigma_Y^2)$ . Define  $\lambda := \sigma_Y^2 / \sigma_X^2$ . Find the level  $\alpha$  LRT of  $H_0 : \lambda = \lambda_0$  versus  $H_1 : \lambda \neq \lambda_0$ .

#### Exercise 33 (Properties of rejection regions)

a) Show that  $\hat{\sigma}_0^2 / \hat{\sigma}^2$  in Exercise 31 can be rewritten as

$$\frac{\hat{\sigma}_0^2}{\hat{\sigma}^2} = 1 + \frac{nm}{n+m} \frac{(\bar{x} - \bar{y})^2}{\hat{\sigma}^2}.$$

b) Express the rejection region of the LRT in Exercise 32 in terms of an  $F(n, m)$  random variable.

*Hint:*  $F$  can be obtained as the ratio of scaled  $\chi^2$  distributions, i.e.  $F(n, m) = \frac{\chi_n^2/n}{\chi_m^2/m}$ .

#### Exercise 34 (Size $\alpha$ test)

Solve the exercise on slide 11 of week 10 regarding to the shifted exponential distribution.

#### Exercise 35 (Most powerful test)

Let  $X_1, \dots, X_n$  be iid random variables from  $Pois(\lambda)$ . Find the most powerful test for  $H_0 : \lambda = 1$  versus  $H_1 : \lambda = 2$ .

**Exercise 36 (Uniform most powerful test)**

Let  $X_1, \dots, X_{10}$  be iid random variables from  $Bernoulli(p)$  with  $p \in [0, 1]$ .

- a) Find the most-powerful test of size  $\alpha = 0.0547$  of the hypotheses  $H_0 : p = 1/2$  versus  $H_1 : p = 1/4$ .
- b) For what  $\alpha$  levels does there exist a UMP test of the previous hypotheses?