

REPORT  
on the  
Research Semester  
GEOMETRIC METHODS IN ANALYSIS  
AND PROBABILITY  
at  
The Erwin Schrödinger International Institute  
for Mathematical Physics  
Vienna

Organisers:

James B. Cooper (Johannes Kepler Universität Linz)

Peter W. Jones (Yale University)

Vitali Milman (Tel Aviv University)

Paul F. X. Müller (Johannes Kepler Universität Linz)

Alain Pajor (Université de Marne-la-Vallée )

David Preiss (University College London)

Carsten Schütt (Christian-Albrechts-Universität Kiel)

Charles P. Stegall (Johannes Kepler Universität Linz)

Period May - August 2005

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# 1 General Remarks

In organising the Research Semester at the Erwin Schrödinger Institute we were guided by the following criteria:

1. It was our intention to choose three themes within the general framework of the mathematical contexts described by the titles which are at present areas of intensive, fruitful and deep research. We also chose topics for which we could be confident that the leading researchers could be attracted in their entirety to Vienna.
2. The themes were chosen to be such that despite their diversity, there was a sufficiency of common ground to allow for (and encourage) interaction and synergetic effects not only within the three groups but also between them.
3. The choice of themes and participants was very carefully arranged with the following aims. Firstly to allow existent joint international research groups (formal and informal) to continue their efforts, secondly to encourage the creation of new groups and thirdly to provide an opportunity for a new and brilliant generation of young mathematicians to participate in these projects. Thus the session on infinite dimensional differentiability should be seen in the context of a long-running series of joint conferences organised twice-annually by Linz and Prague, whereas the meeting on Conformal invariance, Probability and Singular Integrals at the Erwin Schrödinger Institute is part of a series that included the 1999 meeting there and the 2000/2001 research semester at the Mittag Leffler Institute in Djursholm. It will be continued by an International Congress of Mathematicians satellite conference (2006) in Barcelona, organized i.a. by J. Verdera. The section on Asymptotic theory is part of a long term project within the framework of the Euronet project *Phenomena in High Dimensions*.

One important aspect of our organisational work was to supplement the budget provided by the Erwin Schrödinger Institute in order to maximise the use of the available resources. In this we were very successful. The University of Linz and the local government authorities of Upper Austria supplied funds to cover most of the living expenses of the main organizers (thus liberating a large segment of the budget from ESI for the visitors from abroad) plus those of many of the visitors from countries in the former Eastern Block. In addition, they financed the Colloquium at Stift Schlögl. Many of the participants used their own personal grants to prolong their stays in Vienna or to finance those of their students and associates.

An exceedingly important positive input was the parallel organisation of the the **Conference on Convex Geometry and High Dimensional Phenomena** which was subsidised by the European Union Research Training Network (Marie Curie RTN) program *Phenomena in High Dimensions*.

The joint occurrence of the Erwin Schrödinger Institute semester with this conference was a clear example of a case where  $1 + 1$  is not 2 but significantly more since the parallel organisation certainly enhanced the scientific input and output of both conferences enormously.

All of these factors led to an increase of the total funding by about 50 percent.

The extent to which our aims were achieved is, we believe, evident from the formal report below.

The Research Semester was designed to cover the following three themes:

1. GEOMETRIC ASPECTS OF INFINITE DIMENSIONAL BANACH SPACE THEORY, NONLINEAR METHODS IN LINEAR FUNCTIONAL ANALYSIS

The main activities was concentrated in the period May 25<sup>th</sup> — June 5<sup>th</sup>.

Organisers: James Cooper, Linz; David Preiss, London and Charles Stegall, Linz

2. CONFORMAL INVARIANCE, PROBABILITY AND SINGULAR INTEGRALS

The main activities was concentrated in the period June 10<sup>th</sup> — July 5<sup>th</sup>.

Organisers: Peter W. Jones, Yale and Paul F.X. Müller, Linz

3. ASYMPTOTIC THEORY OF THE GEOMETRY OF FINITE DIMENSIONAL SPACES

The main activities was concentrated in the period July 10<sup>th</sup> — August 5<sup>th</sup>.

Organisers: Vitali Milman, Tel-Aviv; Alain Pajor, Paris and Carsten Schütt, Kiel

The main topics were concentration of measure phenomena, transportation of measure, asymptotic theory of convex bodies, geometry of high-dimensional normed spaces.

## 2 Finances

The research semester was supported financially from the budget of the Erwin Schrödinger Institute to the sum of 64.979,57 euros. This covered the living costs of the most of the foreign participants in Vienna. Further financial support was provided by diverse sources and individual grants:

The European Union supported the research semester at the Erwin Schrödinger Institute to the sum of 6.000,- euros and the “Conference on Convex Geometry and High Dimensional Phenomena” to the sum of 3.000,- euros.

The rector of the Johannes Kepler University of Linz provided the costs for 2 flats for the organizers in Vienna (4.943,- euros), the Foreign Office “Auslandsbüro” of the Johannes Kepler University supported the stay of 12 participants in Vienna (4.500,- euros).

Financial support of 2.900,- euros respectively 2.000,- euros for the Colloquium at Stift Schlägl was provided from the Linzer Hochschulfonds and the City Council of Linz “Magistrat der Landeshauptstadt Linz”.

## 3 Scientific Report

### 3.1 Geometric Aspects of Infinite Dimensional Banach Space Theory, Nonlinear Methods in Linear Functional Analysis

The conference under the scientific leadership of D. Preiss (University College London), James B. Cooper and Charles Stegall (Linz) was largely concerned with the development and interaction of the ideas of Preiss and Zajíček about porosity. The ideas concerning porosity have found application in a wide range of areas. In this sense, the conference was particularly fruitful. In particular there were lectures relating porosity to the study of differentiability (Lindenstrauss, Zajíček, Maleva, Duda), the Borel structure of metric spaces (Riss) and convexity (Reich). There were talks in related areas of topology, operator theory, and Banach space geometry. The number of formal lectures was deliberately kept small, so that there would be more opportunities for impromptu lectures and discussions. This was very successful.

We remark that since 1990 the Institute for Analysis in Linz in cooperation with various mathematical institutions in Prague, Bratislava and Salzburg have organized semi-annual regional meetings not only to discuss mathematics, but to improve communications in general, especially regarding students. We think this has been very successful, and this was the second meeting at the Erwin Schrödinger Institute where we extended these regional meetings to broader international congresses. This was an opportunity for workers from the Czech Republic not only to meet with colleagues from Austria but also to meet with colleagues from other parts of the world, particularly Israel and the USA.

Schlumprecht presented his joint work with Odell; there have long been questions related to “universal separable Banach spaces” and Schlumprecht and Odell have shown that there exists a separable Banach space universal for all separable super-reflexive Banach spaces, which seems to be about the best possible result. Fonf and Lindenstrauss presented some of their work concerning James’ theorem. Recently (November, 2005), Kalenda, who attended the conference at ESI, has shown that from a point of view of logic, it appears that Lindenstrauss’ approach to James’ theorem cannot be extended.

#### Participants

| Name                | Institution                          | Period          |
|---------------------|--------------------------------------|-----------------|
| Giovanni ALBERTI    | Università di Pisa                   | 25.05. - 05.06. |
| Miroslav CHLEBIK    | Max Planck Leipzig                   | 25.05. - 05.06. |
| James COOPER        | Johannes Kepler Universität Linz     | 25.05. - 05.06. |
| Jan DUDA            | Univerzita Karlova Praha             | 25.05. - 05.06. |
| Marian FABIAN       | Univerzita Karlova Praha             | 25.05. - 03.06. |
| Vladimir FONF       | Ben Gurion University Be’er Sheva    | 25.05. - 05.06. |
| Petr HÁJEK          | Univerzita Karlova Praha             | 25.05. - 03.06. |
| Petr HOLICKÝ        | Univerzita Karlova Praha             | 27.05. - 03.06. |
| Ondrej KALENDA      | Univerzita Karlova Praha             | 27.05. - 03.06. |
| Hermann KÖNIG       | Christian-Albrechts-Universität Kiel | 20.07. - 29.07. |
| Joram LINDENSTRAUSS | Hebrew Univesity Jerusalem           | 24.05. - 05.06. |

| <b>Name</b>         | <b>Institution</b>               | <b>Period</b>   |
|---------------------|----------------------------------|-----------------|
| Olga MALEVA         | University College London        | 24.05. - 06.06. |
| Jan MALÝ            | Univerzita Karlova Praha         | 27.05. - 04.06. |
| Eva MATOUŠKOVÁ      | Johannes Kepler Universität Linz | 24.05. - 02.06. |
| Vladimir MÜLLER     | Czech Academy of Sciences        | 24.05. - 02.06. |
| Edward ODELL        | University of Texas Austin       | 25.05. - 04.06. |
| David PREISS        | University College London        | 25.05. - 05.06. |
| Simeon REICH        | Technion, Haifa                  | 24.05. - 03.06. |
| Elena RISS          | Steklov Institute St. Petersburg | 25.05. - 05.06. |
| Thomas SCHLUMPRECHT | Texas A&M University             | 25.05. - 10.06. |
| Charles STEGALL     | Johannes Kepler Universität Linz | 25.05. - 05.06. |
| Jaroslav TIŠER      | ČVUT Praha                       | 25.05. - 03.06. |
| Luděk ZAJÍČEK       | Univerzita Karlova Praha         | 27.05. - 05.06. |
| Miroslav ZELENÝ     | Univerzita Karlova Praha         | 25.05. - 03.06. |

### **Talks**

**Jan Duda:** Curves in Banach spaces

**Marian Fabian:** Epsilon-asplund sets and epsilon-weakly compact sets

**Vladimir Fonf:** Around Jame's theorem

**Ondřej Kalenda:** The abstract Dirichlet problem for Baire-one functions

**Joram Lindenstrauss:** Differentiability of Lipschitz functions in Banach spaces and porous sets

**Olga Maleva:** Curves in  $l_1$ -trees and porosity

**Jan Malý:** When difference quotients converge in measure

**Vladimír Müller:** Growth conditions and invertible extensions of operators

**Simon Reich:** Extension theorems, convex functions, and descent methods

**Elena Riss:** On the generation of Borel sets by balls

**Thomas Schlumprecht:** A separable reflexive Banach spaces which is universal for all separable uniformly convex spaces

**Luděk Zajíček:** Sets of non-differentiability and related sigma-porous sets

**Miroslav Zelený:** A note on Buczolich's solution of the Weil problem

## 3.2 Conformal Invariance, Probability and Singular Integrals

The focus of the meeting organized by Peter W. Jones (Yale) and Paul F.X. Müller (Linz) was on the following topics and their interactions

1. Harmonic Measure, Random Fractals and SLE Processes;
2. The Cauchy Integral, Geometric Measure and Rectifiability;
3. Phase Plane Geometry, Multilinear Singular Integrals and Connections to Partial Differential Equations.

The talks of the meeting were given at the the Erwin Schrödinger Institute from Juni 10 — July 5 2005 and a Colloquium was organized at Stift Schögl from June 24 — 26 2005.

Financial support was provided by the Erwin Schrödinger Institute, “Linzer Hochschulfonds” (2.900,- euros) and the City Council of Linz “Magistrat der Landeshauptstadt Linz” (2.000,- euros).

The majority of participants devoted their research activities and talks to the topic

1. Harmonic Measure, Random Fractals and SLE Processes.

These were grouped around the talks and extended problem sessions held by G. Lawler and W. Werner. The talk by G. Lawler was an introduction to SLE (by one of its inventors) while that of W. Werner treated Conformal loop ensembles—an advanced topic in this area. Both problem sessions were aimed at stimulating research activities by younger researchers and graduate students present at the meeting.

The talks by St. Rohde, N. Makarov and N.G Kang covered the relation of SLE processes to random quasi conformal maps (the harmonic explorer and  $SLE_4$ ) Riemann surfaces and Schwarzian Derivatives controlling the boundary behaviour of the conformal maps associated to the SLE processes.

The talks by I. Binder, I. Uriarte and D. Beliaev formed a group discussing Harmonic Measure, random fractals associated to SLE and the multifractal formalism in general.

Peter W. Jones’s talk on RANDOM SHAPES IN NATURE AND MATHEMATICS formed the bracket that held these subjects together and discussed new random phenomena in astrophysics with relations to Harmonic measure, SLE, rectifiability and high dimensional data analysis.

2. The Cauchy Integral, Geometric Measure and Rectifiability

The topics of the Cauchy Integral, Geometric Measure and Rectifiability were centred around one singular integral of particular interest: The Cauchy Integral Operator, its boundedness estimated by the Menger curvature of the underlying measure (X. Tolsa ’s talk) and its point-wise behaviour expressed through the singular support of the underlying measure (talks by J. Verdera, A. Poltoratski and M. Sodin). The talks by K. Astala and M. Gonzalez discussed the relation to the Beltrami Operator and estimates using the geometry of the curves supporting its singular measure. Also motivated by the Beltrami Operator, T. Iwaniec dealt with

pointwise products of  $H^1$  and BMO functions. The connection to extension theorems was the topic of the talks by P. Koskela and L. Rogers (the latter giving simultaneous improvements on both the extension theorem of E.M. Stein and that of P.W. Jones.) A. Nikolau's talk was concerned with interpolation problems of harmonic functions in  $\mathbf{R}^n$ .

Bergman spaces—the topic of H. Hedenmalm's contribution—provide the frame for the functional analytic approach to conformal maps and Cauchy Integrals. They also give the links to Inverse scattering problems (talk by A. Volberg) and Jacobi matrices (talk of P. Yuditskyi).

### 3. Phase Plane Geometry, Multilinear Singular Integrals and Connections to Partial Differential Equations

The presentation by Ch. Thiele—who together with Lacey—invented Phase Plane Geometry in its current complexity—covered trilinear operators, their connection to the Kakeya Problem, group theory and edge detection in image analysis. C. Muscalu—a former student of Ch. Thiele—discussed generalisations of the Carleson Hunt theorem. Phase Plane Geometry and non linear Partial Differential Equations were the topics of the talks by A. Nachmod (non linear Schrödinger operators) and W. Schlag (Stable Manifolds for non linear Partial Differential Equations).

The talks by S. Kislyakov gave simultaneous estimates for Singular Integral Operators under Calderon-Zygmund decompositions. D. Anisimov discussed product forms of Calderon-Zygmund operators and interpolation estimates for square functions. The talks by V. Pillwein and St. Geiss covered rearrangement operators respectively martingale transforms and singular integral operators.

### Participants

| Name                 | Institution                             | Period          |
|----------------------|---|-----------------|
| D. ANISIMOV          | Steklov Institute St. Petersburg        | 14.06. - 24.06. |
| Kari ASTALA          | University of Helsinki                  | 10.06. - 20.06. |
| Imre BARANY          | Hungarian Academy of Sciences           | 12.07. - 28.07. |
| Dimitri BELIAEV      | Royal Institute of Technology Stockholm | 18.06. - 28.06. |
| Ilia BINDER          | University of Illinois                  | 05.06. - 05.07. |
| Alexander I. BUFETOV | Princeton University                    | 20.06. - 30.06. |
| Chris BURDZY         | University of Washington                | 16.06. - 02.07. |
| Stefan GEISS         | University of Jyväskylä                 | ??              |
| Maria GONZALES       | Universidad de Cádiz                    | 10.06. - 20.06. |
| Hakan HEDENMALM      | Lunds Universitet                       | 25.06. - 05.07. |
| Tadeusz IWANIEC      | Syracuse University                     | 23.06. - 30.06. |
| Peter W. JONES       | Yale University                         | 05.06. - 29.06. |
| Nam-Gyu KANG         | Massachusetts Institute of Technology   | 09.06. - 05.07. |
| Sergiu KISLIAKOV     | St. Petersburg State University         | 09.06. - 18.06. |
| Boaz KLARTAG         | IAS Princeton                           | 10.07. - 05.08. |
| Pekka KOSKELA        | University of Jyväskylä                 | 13.06. - 17.06. |
| Gregory F. LAWLER    | Cornell University                      | 09.06. - 16.06. |
| Nikolai MAKAROV      | Caltech Pasadena                        | 24.06. - 02.07. |

| Name                   | Institution                       | Period          |
|------------------------|-----------------------------------|-----------------|
| Mark MELNIKOV          | Universitat Autònoma de Barcelona | 12.06. - 19.06. |
| Paul MÜLLER            | Johannes Kepler Universität Linz  | 05.06. - 05.07. |
| Camil MUSCALU          | Cornell University                | 10.06. - 03.07. |
| Andrea NAHMOD          | University of Massachusetts       | 26.06. - 06.07. |
| Artur NICOLAU          | Universitat Autònoma de Barcelona | 12.06. - 18.06. |
| Vladimir PELLER        | Michigan State University         | 15.06. - 05.07. |
| Veronika PILLWEIN      | Johannes Kepler Universität Linz  | 20.06. - 24.06. |
| Alexei POLTORATSKI     | Texas A&M University              | 20.06. - 04.07. |
| Luke ROGERS            | Cornell University                | 09.06. - 06.07. |
| Steffen ROHDE          | University of Washington          | 13.06. - 26.06. |
| Wilhelm SCHLAG         | Caltech Pasadena                  | 01.07. - 07.07. |
| Raanan SCHUL           | Yale University                   | 10.06. - 5.07.  |
| Andreas SEEGER         | University of Wisconsin-Madison   | 19.06. - 24.06. |
| Misha SODIN            | Tel Aviv University               | 22.06. - 02.07. |
| Christoph THIELE       | UCLA                              | 24.06. - 03.07. |
| Xavier TOLSA           | Universitat Autònoma de Barcelona | 12.06. - 23.06. |
| Sergei TREIL           | Michigan State University         | 26.06. - 07.07. |
| Ignacio URIARTE-TUERO  | University of Helsinki            | 09.06. - 06.07. |
| Joan VERDERA           | Universitat Autònoma de Barcelona | 10.06. - 15.06. |
| Alexander VOLBERG      | Michigan State University         | 14.06. - 04.07. |
| Wendelin WERNER        | Université Paris Sud              | 25.06. - 05.07. |
| Name                   | Institution                       | Period          |
| Michał WOJCIECHOWSKI   | Polish Academy of Sciences        | 25.05. - 05.06. |
| Przemysław WOJTASZCZYK | Polish Academy of Sciences        | 04.06. - 19.06. |
| Peter YUDITSKYI        | Johannes Kepler Universität Linz  | 23.06. - 02.07. |

### Talks

- June  
**Michał Wojciechowski:** Bounded approximation property of the functions of bounded variation of higher order
- June 10, 11:00  
**Serguei Kislyakov:** Stability of approximation under singular integral operators, and Calderon-Zygmund type decomposition (Joint work with Natan Kruglyak)
- June 13, 14:00  
**Przemysław Wojtaszczyk:** 1-greedy bases
- June 13, 14:00  
**Joan Verdera:** May the Cauchy transform of a non trivial finite measure vanish on its support?
- June 13, 16:00  
**Artur Nicolau:** An interpolation problem for positive harmonic functions

- June 15, 11:00  
**Greg Lawler:** Introduction to SLE Processes
- June 15, 16:00  
**Greg Lawler:** Problem session on SLE Processes
- June 16, 14:00  
**Xavier Tolsa:** Cauchy integrals of measures and rectifiability
- June 16, 16:00  
**Maria Gonzalez:** Geometry of curves and Beltrami-type Operators (Joint work with K. Astala)
- June 17, 11:00  
**Kari Astala:** Improved Painleve removability for  $K$ -quasiregular mappings
- June 17, 14:00  
**Pekka Koskela:** Sobolev extension theorems
- June 20, 11:00  
**Raanan Schul:** A characterization of subsets of finite length curves in Hilbert spaces
- June 20, 16:00  
**Luke Rogers:** Degree independent Sobolev Extension
- June 21, 11:00  
**Ilia Binder:** Multifractal analysis of harmonic measure
- June 21, 16:00  
**Nam-Gyu Kang:** Boundary behaviour for SLE
- June 22, 14:00  
**Alexei Poltoratski:** Extensions and Applications of Beurling-Malliavin theory
- June 23, 11:00  
**D. Beliaev:** Harmonic measure on random fractals
- June 23, 14:00  
**D. Anisimov:** An interpolation problem involving double singular integrals
- June 23, 16:00  
**Ignatio Uriarte-Tuerto:** On Marcinkiewicz integrals and harmonic measure
- June 25, 9:00  
**Peter W. Jones:** Random shapes in nature and mathematics
- June 25, 10:00  
**Steffen Rohde:** Random quasi conformal maps
- June 25, 11:00  
**Chris Burdzy:** Robin problem and the Ray Knight theorem

- June 25, 19:30  
**Tadeusz Iwaniec:** The product of  $H_1$  and  $BMO$  functions
- June 26, 9:00  
**Camil Muscalu:** A generalization of the Carleson Hunt theorem
- June 26, 9:55  
**Misha Sodin:** Random complex zeros
- June 26, 10:30  
**Stefan Geiss:** Vector valued martingale transforms
- June 26, 11:45  
**Vladimir Peller:** Analytic approximation of rational matrix functions
- June 27, 14:00  
**Piotr Yuditskiy:** On generalized sum rules for Jacobi matrices
- June 27, 16:00  
**Nikolai Makarov:** SLE on Riemann surfaces
- June 28, 11:00  
**Wendelin Werner:** Conformal loop ensembles
- June 28, 16:00  
**Wendelin Werner:** Problem session on SLE
- June 29, 11:00  
**Veronika Pilwein:** Extrapolation of Rearrangement Operators
- June 29, 16:00  
**Christoph Thiele:** Trilinear Maximal Function
- July 1, 11:00  
**Hakan Hedenmalm:** Introduction to Bergmann spaces with applications to conformal maps
- July 4, 11:00  
**Alexander Volberg:** Inverse scattering and matrix A2
- July 4, 16:00  
**Andrea Nahmod:** Non linear Schrödinger equations
- July 5, 11:00  
**Wilhelm Schlag:** On stable manifolds for certain non-linear Partial Differential Equation

## Participants Colloquium at Stift Schlägl

| Name           | Institution                             |
|----------------|---|
| D. Beliaev     | Royal Institute of Technology Stockholm |
| I. Binder      | University of Illinois                  |
| C. Burdzy      | University of Washington                |
| S. Geiss       | University of Jyväskylä                 |
| T. Iwaniec     | Syracuse University                     |
| P. Jones       | Yale Univeristy                         |
| N.G. Kang      | Massachusetts Inst.of Technology        |
| C. Muscalu     | Cornell University                      |
| V. Peller      | Michigan State University               |
| V. Pillwein    | Johannes Kepler Universität Linz        |
| A. Poltoratski | Texas A&M University                    |
| L. Rogers      | Cornell University                      |
| C. Rohde       | University of Washington                |
| R. Schul       | Yale University                         |
| M. Sodin       | Tel Aviv University                     |
| I. Uriarte     | University of Helsinki                  |
| P. Yuditskyi   | Johannes Kepler Universität Linz        |
| Organizer:     |   |
| P. Müller      | Johannes Kepler Universität Linz        |
| R. Mühlbachler | Johannes Kepler Universität Linz        |

## Talks Colloquium at Stift Schlägl

### Saturday, June 25, 2005

- 09:00—09:50 P. Jones  
Random Shapes in Nature and Mathematics
- 10:00—10:50 S. Rohde  
Random Quasi Conformal Maps
- 11:20—12:10 C. Burdzy  
Robin Problem and the Ray Knight theorem
- 19:30—20:15 T. Iwaniec  
The product of  $H_1$  and  $BMO$  functions

### Sunday, June 26, 2005

- 09:00—09:45 C. Muscalu  
A Generalization of the Carleson Hunt Theorem
- 09:55—10:30 M. Sodin  
Random Complex Zeros
- 10:50—11:30 S. Geiss  
Vector Valued Martingale Transforms
- 11:45—12:30 V. Peller  
Analytic Approximation of Rational

### 3.3 Asymptotic Theory of the Geometry of Finite Dimensional Spaces

The main topics were concentration of measure phenomena, transportation of measure, asymptotic theory of convex bodies and the geometry of high-dimensional normed spaces. This part of the workshop took place from July 11, 2005 till August 5, 2005.

The workshop was structured in the following way. The first 10 days took place at the Schrödinger Institute, after which a conference of 8 days followed which was organized at the Technische Universität. The last 10 days of the workshop took place at the Schrödinger Institute.

The organizers V. Milman, A. Pajor and C. Schütt are part of the Research Training Network (Marie Curie RTN) program *Phenomena in High Dimensions* (PHD) funded by the European Union. The topics of the workshop were also topics of PHD. The European Union supported the workshop through RTN to the sum of 6 000,- euros.

Our colleagues at the Technische Universität P. Gruber, M. Ludwig and M. Reitzner are also part of RTN and the conference which we organized jointly with them received 3 000,- euros support from RTN. There were 115 participants at the conference.

During the first and last 10 days of the workshops usually we had 2 or 3 introductory talks with two functions: firstly, as it had been important to us to invite young people (out of the 65 people who participated at the workshop there were 20 young researchers or post docs), they were directed toward those colleagues. Secondly, these talks were intended to draw attention to problems and ideas that have not been explored sufficiently in the past. During the conference, a number of scientific talks in which the leading researchers in this field presented their latest results were organised. Some time was made available for young researchers.

Many of the participants of the workshop used grants from their home institutions or their national science foundations to support a longer stay at the Schrödinger Institute or to support their students.

Concerning new research let us mention here some of the progress. A basic open problem in asymptotic convex geometry is the so-called slicing problem. One of its many interesting formulations is the following question: What is the minimal number  $L_n$ , such that any  $n$ -dimensional convex set of volume one contains a hyperplane section of volume larger than  $1/L_n$ ? This problem has attracted a large amount of attention in recent decades. Until recently, the best estimate has been that of Bourgain (1984):  $L_n < cn^{1/4} \log n$ . A participant of the workshop, Klartag, has succeeded in improving this long-standing estimate. His result is that  $L_n < cn^{1/4}$ . Furthermore, he was able to show that convex bodies with a bounded isotropic constant constitute an  $\epsilon$ -net in the Banach Mazur compactum, in the following sense: For any  $n$ -dimensional convex set  $K$ , and  $\epsilon > 0$ , there is another convex body  $T$  whose distance from  $K$  is at most  $\epsilon$  and the isotropic constant of  $T$  is bounded by a function that depends solely on epsilon, and not on the dimension. This provides a solution to a weak version of the slicing problem, which is known as the isomorphic slicing problem.

In the course the workshop discussions were held that led to the following joint work of Milman and Artstein-Avidan: The authors show how to decrease the amount of randomness needed to achieve some of the basic geometric constructions in asymptotic convex geometry. For example, it was shown how an  $n/2$ -dimensional section of the  $\ell_1^n$  ball which is isomorphic to the euclidean ball, well known to exist probabilistically, when choosing the subspace as a

kernel of  $n/2$  random independent sign vectors, can be constructed by choosing only  $\log n$  random sign vectors. Similar results on randomness reduction were shown by the authors to hold true ( $QS$  theorems, low  $M^*$  estimates, ZigZag bodies and global Dvoretzky's theorem). In most examples a derandomization technique from computer science is exploited, imposing an expander on the set of sign vectors with a relatively small degree and taking a random walk on this expander (this tool was first used by Ajtai Komlos and Szemerédi). Some more delicate tools were developed since the theory did not exactly fit the needs of the geometric proofs. In particular a derandomized version of Bernstein's inequality which allows one to disregard a small amount of the vectors was proved by the authors.

The participant Paouris obtained the following result. A convex body  $K$  in  $\mathbf{R}^n$ , with volume equal to 1 and center of mass at the origin, is called isotropic if its inertia matrix is a multiple of the identity. Equivalently, if there exists a positive constant  $L_K$  such that  $\int_K \langle x, \theta \rangle^2 dx = L_K^2$  for every  $\theta \in S^{n-1}$ . The starting point of this paper is the following concentration estimate of Alesker: If  $K$  is an isotropic convex body in  $\mathbf{R}^n$  then, for every  $t \geq 1$  we have

$$\text{Prob} \left( \{x \in K : \|x\|_2 \geq c\sqrt{n}L_K t\} \right) \leq 2 \exp(-t^2).$$

Bobkov and Nazarov have obtained an unexpected strengthening of Alesker's estimate for the class of 1-unconditional isotropic convex bodies: in this case,

$$\text{Prob} \left( \{x \in K : \|x\|_2 \geq c\sqrt{n}L_K t\} \right) \leq \exp(-\sqrt{nt})$$

for every  $t \geq 1$ . Paouris proved the striking result that the "Bobkov-Nazarov estimate" holds true in full generality.

An application of this result concerns a question of Kannan, Lovász and Simonovits which has its origin in the problem of finding a fast algorithm for the computation of the volume of a given convex body. Let  $\varepsilon \in (0, 1)$  and consider  $N$  independent random points  $x_1, \dots, x_N$  uniformly distributed in an isotropic convex body  $K$  in  $\mathbf{R}^n$ . The question is to find  $N_0$ , as small as possible, for which the following holds true: if  $N \geq N_0$  then with probability greater than  $1-\varepsilon$  one has  $\|I - \frac{1}{NL_K^2} \sum_{i=1}^N x_i \otimes x_i\| \leq \varepsilon$ . Bourgain proved that one can choose  $N_0 \simeq c(\varepsilon)n(\log n)^3$ ; this was improved to  $N_0 \simeq c(\varepsilon)n(\log n)^2$  by Rudelson. The previous result allows Paouris to remove one more logarithmic term.

Both results remain valid if one replaces the Lebesgue measure on an isotropic convex body by an arbitrary isotropic log-concave measure.

The following list includes the participants at the Erwin Schrödinger Research Semester and the conference on Convex Geometry at the Technical University.

### Participants

| <b>Name</b>     | <b>Institution</b> | <b>Period</b> |
|-----------------|--------------------|---------------|
| S. Alesker      | Tel Aviv           | 20.7.-27.7.   |
| S. Artstein     | Princeton          | 11.7.-31.7.   |
| G. Aubrin       | Paris, Athens 6    | 11.7.-5.8.    |
| K. Ball         | London, Redmond    | 20.7.-27.7.   |
| I. Barany       | Budapest, London   | 12.7.-27.7.   |
| F. Barthe       | Tolouse            | 13.7.-27.7.   |
| J. Bastero      | Zaragoza           | 11.7.-27.7.   |
| K. Böröczky     | Budapest           |               |
| Y. Brenier      | Nice               | 20.7.-27.7.   |
| C. Buchta       | Salzburg           | 11.7.-19.7.   |
| D. Cordero      | Marne la Vallee    | 18.7.-1.8.    |
| A. Dmitriyuk    |                    | 11.7.-5.8.    |
| M. Fradelizi    | Marne-la-Vallé     | 20.7.-27.7.   |
| O. Friedland    | Tel Aviv           | 11.7.-5.8.    |
| D. Gatzouras    | Iraklio            | 18.7.-27.7.   |
| A. Giannopoulos | Athens             | 18.7.-27.7.   |
| E. Gluskin      | Tel Aviv           | 14.7.-27.7.   |
| Y. Gordon       | Haifa              | 10.7.-5.8.    |
| P. Gruber       | Wien               |               |
| O. Guédon       | Paris 6            | 10.7.-31.7.   |
| A. Hinrichs     | Jena               | 11.7.-27.7.   |
| D. Hug          | Freiburg           | 25.7.-3.8.    |
| G. Kalai        | Jerusalem          | 15.7.-1.8.    |
| B. Klartag      | Tel Aviv           | 11.7.-5.8.    |
| H. König        | Kiel               | 20.7.-29.7.   |
| A. Koldobsky    | Columbia-Missouri  | 11.7.-5.8.    |
| Krivelevich     | Tel Aviv           | 22.7.-29.7.   |
| R. Latała       | Warsaw             | 19.7.-3.8.    |
| L. Lovasz       | Budapest, Redmod   | 20.7.-27.7.   |
| M. Ludwig       | Wien               |               |
| P. Mankiewicz   | Warsaw             | 20.7.-5.8.    |
| Matousek        | Prag               | 20.7.-27.7.   |
| M. Meckes       | Stanford           | 20.7.-27.7.   |
| M. Meyer        | Marne-la-Vallé     | 11.7.-1.8.    |
| E. Milman       | Rehovot            | 11.7.-5.8.    |
| V. Milman       | Tel Aviv           | 11.7.-5.8.    |
| K. Oleszkiewicz | Warsaw             | 19.7.-3.8.    |
| A. Pajor        | Marné-la-Vallée    | 11.7.-31.7.   |
| G. Paouris      | Paris              | 11.7.-5.8.    |
| L. Pastur       | Kharkov            | 20.7.-29.7.   |

| <b>Name</b>           | <b>Institution</b>        | <b>Period</b> |
|-----------------------|---------------------------|---------------|
| A. Pelczynski         | Warsaw                    | 16.7.-31.7.   |
| G. Pisier             | Paris, Texas              | 20.7.-27.7.   |
| P. Pivovarov          | Edmonton                  | 9.7.-5.8.     |
| S. Reisner            | Haifa                     | 10.7.-1.8.    |
| M. Reitzner           | Wien                      |               |
| M. Rudelson           | Columbia-Missouri         | 11.7.-27.7.   |
| D. Ryaborgin          | Kansas State University   | 27.7.-5.8.    |
| I. Ryzhkova           | Kharkov                   | 20.7.-27.7.   |
| M. Saß                |                           | 10.7.-18.7.   |
| M. Schmitz            |                           | 10.7.-18.7.   |
| M. Schmuckenschläger  | Linz                      |               |
| R. Schneider          | Freiburg                  | 12.7.-28.7.   |
| C. Schütt             | Kiel                      | 10.7.-5.8.    |
| A. Sodin              | Tel Aviv                  | 11.7.-5.8.    |
| A. Stancu             | Montreal                  | 31.7.-5.8.    |
| S. Szarek             | Paris                     | 12.7.-27.7.   |
| A. Shcherbina         | Kharkov                   | 20.7.-27.7.   |
| M. Shcherbina         | Kharkov                   | 20.7.-30.7.   |
| N. Tomczak-Jaegermann | Edmonton                  | 10.7.-5.8.    |
| T. Tsesmetzis         |                           | 15.7.-31.7.   |
| A. Tsolomitis         | Mitilene                  | 18.7.-27.7.   |
| R. Vershynin          | Davis, CA                 | 11.7.-5.8.    |
| R. Vitale             | University of Connecticut | 15.7.-28.7.   |
| R. Wagner             |                           | 12.7.-19.7.   |
| W. Weil               | Karlsruhe                 | 15.7.-29.7.   |
| E. Werner             | Cleveland                 | 10.7.-5.8.    |
| V. Yaskin             | Columbia-Missouri         | 11.7.-5.8.    |

### **The Erwin Schrödinger Talks**

#### **Tuesday, July 12**

14:00 P.M. Gruber (Wien): *Principles of classical discrete geometry*

15:30 S. Artstein (Princeton): *Metric entropy and coverings-duality*

#### **Wednesday, July 13**

11:00 I. Barany (Budapest and London): *On the power of linear dependencies*

14:00 C. Buchta (Salzburg): *What is the number of vertices of the convex hull of  $N$  randomly chosen points?*

15:30 K. Böröczky (Budapest): *Stability of affine invariant geometric inequalities*

#### **Thursday, July 14**

11:00 B. Klartag (Clay Institute): *Diameters of sections of convex bodies*

14:00 A. Koldobsky (Columbia-Missouri):

15:30 F. Barthe (Toulouse): *Entropy of spherical marginals*

#### **Friday, July 15**

14:00 W. Weil (Karlsruhe): *Boolean models and convexity*

15:30 R. Schneider (Freiburg): *Simplices I*

**Monday, July 18**

14:00 R. Schneider (Freiburg): *Simplices II*

15:30 N. Tomczak-Jaegermann (Edmonton): *Decoupling weakly dependent events*

**Tuesday, July 19**

14:00 A. Giannopoulos (Athens): *Random 0 – 1-polytopes*

15:30 G. Kalai (Jerusalem): *Fourier analysis of Boolean functions*

**Thursday, July 28**

11:00 L. Pastur (Kharkov): *A simple approach to the global regime of random matrix theory*

14:00 H. König (Kiel): *Spherical functions and Grothendieck's inequality*

15:30 M. Shcherbina (Kharkov): *Universality of local eigenvalue statistics for matrix models*

**Friday, July 29**

14:00 D. Cordero-Erausquin (Marne la Vallee):  *$L^2$ -methods for Prekopa's theorem*

15:30 L. Pastur (Kharkov): *A simple approach to the global regime of random matrix theory*

**Monday, August 1**

14:00 R. Latała (Warsaw): *On majorizing measures*

15:30 K. Oleszkiewicz (Warsaw): *Kwapień's theorem*

**Thursday, August 4**

14:00 A. Stancu (Montreal): *Floating bodies*

For the completeness we include a list of talks given at the

**Conference on Convex Geometry and High Dimensional Phenomena**

**Wednesday, July 20**

10:15-11:05 S. Alesker (Tel Aviv): *Theory of valuations on manifolds*

11:25-12:15 B. Klartag (Tel Aviv): *From isomorphic to almost-isometric problems in asymptotic convex geometry*

14:00-14:20 Z. Füredi (Budapest and Illinois): *Sets of few distances in high dimensional normed spaces*

14:30-14:50 Y. Gordon (Haifa): *Probabilistic min-max theorems revisited and applications to geometry*

15:00-15:20 R. Latała (Warsaw): *Moments and tail estimates for Gaussian chaoses*

15:30-15:50 H. Vogt (Dresden): *Central limit theorems in the  $W_2^k$ -norm for one-dimensional marginal distributions*

16:30-16:50 O. Guédon (Paris):  *$L_p$  moments of random vectors via majorizing measure*

17:00-17:20 J. Bastero (Zaragoza): *Upper estimates for the volume and the diameter of sections of symmetric convex bodies*

**Thursday, July 21**

9:00-9:50 L. Pastur (Kharkov): *Limiting laws of fluctuations of linear eigenvalue statistics of matrix models*

- 10:10-11:00 S. Szarek (Paris and Cleveland): *Tensor products of convex sets*
- 11:30-11:50 R. Vershynin (Davis,CA): *Signal processing: geometric and probabilistic perspectives*
- 12:00-12:30 M. Shcherbina (Kharkov): *Double scaling limit for matrix models with non analytic potential*
- 14:00-14:20 P. Salani (Florence): *A Brunn-Minkowski inequality for the Monge-Ampere eigenvalue*
- 14:30-14:50 M. Meckes (Stanford): *The central limit problem for random vectors with symmetries*
- 15:00-15:20 E. Meckes (Stanford): *Normal approximation under continuous symmetries*
- 15:30-15:50 A. Hinrichs (Jena): *Optimal geometric design of high-dimensional cubature formulas*
- 16:30-16:50 K. Marton (Budapest): *Logarithmic Sobolev inequality for weakly dependent spin systems*
- 17:00-17:20 I. Ryshkova (Kharkov): *Nonlinear oscillation of a plate in a potential gas flow in the presence of thermal effects*
- 17:30-17:50 A.S. Shcherbina (Kharkov): *Solutions of dissipative Zakharov systems*

### Friday, July 22

- 9:00-9:50 I. Barany (Budapest and London): *Recent results on random polytopes*
- 10:10-11:00 R. Schneider (Freiburg): *Limit shapes in random mosaics and isoperimetric inequalities*
- 11:30-11:50 A. Koldobsky (Columbia, MO): *On the road from intersection bodies to polar projection bodies*
- 12:00-12:30 A. Giannopoulos (Athens): *Asymptotic formulas for proportional sections of convex bodies*
- 14:00-14:20 K. Böröczky Jr. (Budapest): *Approximation of smooth convex bodies by circumscribed polytopes with respect to the surface area*
- 14:30-14:50 C. Peri (Milan): *Discrete tomography: point x-rays of convex lattice sets*
- 15:00-15:20 G. Bianchi (Florence): *The covariogram of 2-,3- and 4-dimensional convex polytopes*
- 15:30-16:00 M.A. Hernandez (Cifre): *The Steiner polynomial and a problem by Hadwiger*
- 16:30-16:50 K. Bezdek (Budapest and Calgary): *On the illumination parameters of smooth convex bodies*
- 17:00-17:20 B.V. Dekster (New Brunswick): *The total angle around a point in Minkowski plane*
- 17:30-17:50 J.M. Aldaz (Rioja): *Behavior of the maximal function in high dimensions*

### Monday, July 25

- 9:00-9:50 K.M. Ball (London and Redmond): *Markov type and the non-linear Maurey extension theorem*
- 10:00-10:20 A. Colesanti (Florence): *A functional inequality related to the Rogers-Shephard inequality*
- 10:30-10:50 M. Fradelizi (Marne-la-Vallée): *On some functional forms of Santaló inequality*
- 11:30-11:50 G. Paouris (Paris): *Concentration of mass on the Schatten classes*
- 12:00-12:20 N. Markoulakis (Heraklion):  *$-1/1$  polytopes with many faces*
- 14:00-14:20 F. Schuster (Wien): *Geometric inequalities for rotation equivariant additive mappings*

14:30-14:50 P. Pivovarov (Edmonton): *A convex body lacking symmetric projections*  
15:00-15:20 V. Yaskin (Columbia-Missouri): *The Busemann-Petty problem in hyperbolic and spherical spaces*

**Tuesday, July 26**

9:00-9:50 F. Barthe (Toulouse): *Functional approach to isoperimetry and concentration*  
10:10-11:00 G. Pisier (Paris): *Similarity problems and amenability for groups and operator algebras*  
11:30-11:50 J. Matousek (Prague): *Challenges of combinatorial linear programming*  
12:00-12:20 G. Aubrun (Paris and Athens): *Sampling convex bodies: a random matrix approach*  
14:00-14:20 V. Vengerovskiy (Tel Aviv): *Eigenvalue distribution of some ensembles of sparse random matrices*  
14:30-14:50 T. Müller (Budapest): *The chromatic number of random geometric graphs*  
15:00-15:20 E. Milman (Rehovot): *Using dual mixed volumes to bound the isotropic constant*  
16:00-16:20 G. Averkov (Chemnitz and Florence): *Convex bodies with critical cross-section measures*  
16:30-16:50 M. Naszódi (Calgary): *Ball-polytopes in Euclidean spaces*

**Wednesday, July 27**

9:00-9:50 Y. Brenier (Nice): *Optimal transportation of currents*  
10:10-11:00 L. Lovasz (Budapest and Redmond): *Graph limits, Szemerédi's regularity lemma and some Banach spaces*  
11:30-11:50 M. Krivelevich (Tel Aviv): *Smoothed analysis in graphs and Boolean formulas*  
12:00-12:20 G. Kalai (Jerusalem): *Noise sensitivity and noise stability, some recent results*  
12:30-12:50: A. Pajor (Marne-la-Vallée): *Reconstruction and subgaussian operators*