Sudden Stop: Supply and Demand Shocks in the German Natural Gas Market*

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Abstract

We propose a structural vector-autoregressive model for the natural gas market to investigate the impact of the 2022 Russian supply stop on the German economy. We combine conventional and narrative sign restrictions to leverage information about supply cuts for identification and find that supply and demand shocks have large and persistent price effects, whereas output effects are rather short-lived. The 2022 natural gas price spike was driven by negative supply and positive storage demand shocks. Counterfactual simulations of an earlier embargo on Russian gas imports indicate only moderately larger negative output effects compared to what we observe in the data.

JEL codes: E32, F51, Q41, Q43, Q48
Keywords: Energy crisis, German natural gas market, narrative sign restrictions, natural gas price, structural scenario analysis, vector-autoregression

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1 Introduction

The level of natural gas prices in the European Union surged in the summer of 2021, ending a two-decade period of low and stable prices. Both demand pressures, stemming from the economic recovery following the COVID-19 pandemic and supply disruptions related to the Russian invasion of Ukraine arguably contributed to this unexpected development. At the peak of the crisis, the price of one-month ahead natural gas futures for the European market (TTF) had risen by a factor twelve relative to its 2019 average (see Figure 1). While import prices, which are more relevant for industry, do not co-move perfectly with gas futures prices, they also started to increase substantially in Germany from 2021 onward.

Figure 1 also highlights the regional segmentation, which distinguishes natural gas markets from the global market for crude oil. While natural gas prices surged in Europe, they exhibited a much smaller increase in East Asia (JKM) and remained relatively stable in the US (Henry Hub). Despite recent political attempts to launch liquefied natural gas (LNG) terminals along the German coastline, time to build and limited global capacity of LNG maritime vessels suggests that natural gas markets are likely to remain segmented for the foreseeable future, with pipeline transport as the dominant mode of transportation.[1]

Analyzing price dynamics in regional natural gas markets thus helps to better understand the cause of regional business cycles. Accordingly, this paper proposes a structural vector-autoregressive (SVAR) model of the German natural gas market to investigate the effects of natural gas supply and demand shocks on domestic gas supply, import prices, inventories, and aggregate economic activity. We consider Germany as a particularly interesting case. In its efforts to phase out coal and nuclear energy from the energy mix, Germany decided to rely predominantly on natural gas, until renewable energy sources are sufficient to cover domestic demand. Before 2022, more than half of Germany's supply of natural gas was imported from Russia, suggesting a strong energy dependency. Moreover, the German economy is characterized by a comparatively high value-added share of industry, a significant fraction of which can be classified as energy-intensive.[2] For these reasons, Germany appears to be particularly vulnerable to disruptions in the natural gas market.

To investigate the drivers and economic consequences of the recent surge in natural gas prices, we draw on the extensive literature on SVAR models of the global oil market and distinguish between structural natural gas supply and demand shocks by imposing sign restrictions on impulse response functions. To sharpen inference, we complement these assumptions with additional narrative restrictions on the sign, size, or effects of shocks during well-documented episodes in 2022 as well as an earlier natural gas supply

[1] For a detailed analysis regarding the obstacles to natural gas trade, see Barbe and Riker (2015).

[2] In 2022, the value-added share of German industry was 25%, 16% of which was classified as energy-intensive.

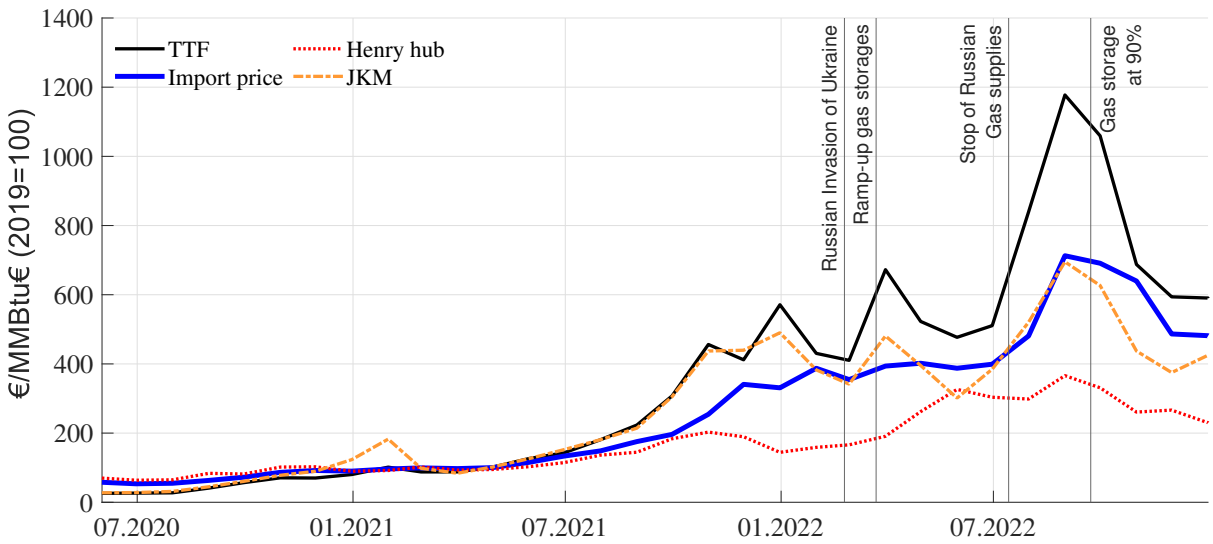

Figure 1: German natural gas import price and one-month ahead natural gas futures for Europe (TTF), the US (Henry Hub), and northeast Asia (JKM)

shock associated with the Russia-Ukraine gas transit dispute in 2009. Our econometric framework thus allows us to quantify the contribution of each of these shocks to fluctuations in domestic natural gas prices and economic activity — both on average over the sample period and during the recent natural gas price surge.

The structurally shocks explain up to 90% of the variance in the endogenous variables, suggesting that the model successfully captures the key structural drivers of the German natural gas market. We find that supply and demand shocks in the natural gas market have large and persistent price effects, albeit moderate output effects. Regarding the 2022 energy crisis, we find that negative supply and positive storage demand shocks contributed disproportionately to the surge of natural gas import prices between February and August 2022.[3] Nevertheless, the impact on German industry was moderate. Despite large adverse natural gas supply shocks, industrial production remained fairly robust. After the natural gas price spike in the summer of 2022, the relatively mild winter that followed led to an easing of domestic natural gas prices and about 20% higher natural gas inventories compared to what would have been expected in an average winter. This easing of the natural gas market occurred despite lower natural gas imports, as increases in imports from Norway, Belgium and the Netherlands did not fully compensate for the lack of imports from Russia.

An immediate disruption of gas imports from Russia in April 2022 — for example due to a German embargo on Russia demanded by some politicians and economists — would likely have led to only moderately and temporarily higher gas import prices compared to the actual scenario, in which flows through the Nord Stream 1 pipeline connecting Russia

[3]Figure A.1 in Appendix A.1 illustrates the successful efforts by the German government to ramp up gas inventories, following exceptionally low levels in March 2022. The political aim to increase the level of gas inventories to 90% by November was already reached in October 2022.

and Germany were reduced to zero in three steps between June and September of 2022. A hypothetical disruption of natural gas flows through Europipe 1 and 2, which transport natural gas from Norway to Germany and currently account for 36% of German imports, is predicted to have comparable effects on natural gas import prices and aggregate economic activity.[4] A key assumption for these counterfactual simulations is that substitution patterns remain similar to those observed in the summer of 2022. The effects should therefore be interpreted as a lower bound. Given that Germany currently relies on only three main natural gas suppliers — Norway, Belgium, and the Netherlands — the loss of another supplier could be more difficult to compensate, as many pipelines to Germany are operating at close-to-full capacity. Although Germany has recently commissioned LNG terminals, the amount of natural gas imported via these terminals has so far been negligible and is unlikely to significantly increase the scope for substitution, at least in the short term.

Our work contributes to the recent policy debate on the macroeconomic effects of energy price shocks in import-dependent economies, such as Germany, and the potential output losses of an embargo on natural gas imports from Russia (see, e.g., Bachmann et al., 2022; German Council of Economic Experts, 2022; Krebs, 2022). Methodically, we build on the extensive SVAR literature studying the market for crude oil. Starting with Kilian (2009), numerous contributions have disentangled the effects of supply and demand shocks in the global oil market on the price of crude oil and thus on economic conditions in the U.S. and abroad (see, in particular, Kilian and Murphy, 2012, 2014; Baumeister and Peersman, 2013a,b; Baumeister and Hamilton, 2019). Despite the recent commercialization of techniques to liquefy natural gas for transport using LNG maritime vessels, regional gas markets remain comparatively fragmented, as illustrated by substantial natural gas price differentials between the U.S., Asia, and continental Europe in Figure 1. In contrast to the global oil market, our analysis is therefore tailored to a specific regional gas market. Furthermore, the effect of temperature is much more important in the natural gas market, as natural gas is the most important energy source for heating in Germany.

The SVAR analysis in this paper focuses on the natural gas market of Germany — the world's fourth largest economy, which is highly dependent on imports of primary energy carriers — to disentangle the effects of natural gas supply and demand shocks using identifying strategies that are well established in the oil-market literature. It is thus related to previous work by Nick and Thoenes (2014), who also investigate the role of supply and demand shocks in the German natural gas market in a recursively identified SVAR model à la Kilian (2009), albeit lacking the recent episode for identification.[5]

[4] Europipe 1 and 2 are delivering natural gas from the Norwegian Draupner E platform to Dornum and from Kårstø in Norway to Emden in Germany, respectively. Along the German coast, the two pipelines run next to each other in shallow water, making them susceptible to targeted disruptions.

[5] Using market-based measures of inflation expectations, Böck and Zörner (2023) instead focus on the

The rest of this paper is structured as follows. Section 2 discusses the SVAR model, data, and identifying assumptions as well as the importance of narrative sign restrictions. Section 3 presents and discusses our baseline results. Section 4 conducts scenario analyses to quantify the effects of a hypothetical Russian gas embargo and different temperature paths during 2022. Section 5 presents the robustness checks, while Section 6 concludes.

2 Empirical Methodology

In this section we present the econometric model and the time series used for estimation.

2.1 Model

We model the dynamics of the German market for natural gas using a four-variable SVAR:

$$A_0 y_t = c + \sum_{l=1}^{12} A_l y_{t-l} + \sum_{i=1}^{11} \gamma_i s_i + \delta x_t + \epsilon_t, \qquad \epsilon_t \sim N(0, I_n), \tag{1}$$

where the $n \times 1$ vector of endogenous variables y_t contains the percentage change in domestic gas supply, an indicator of real economic activity, the percentage change of the real gas price, and the change in domestic gas inventories. We account for seasonal variation in the seasonally unadjusted time series by including monthly dummies s_i, which equal one for the respective month and zero otherwise.[6] Moreover, we include the average monthly temperature, x_t, as an exogenous regressor to control for temperature-related gas demand, in particular for consumption in gas-heating systems. We include an entire year of lags to allow for persistent cycles in the German natural gas market.

Given that $u_t = A_0^{-1}\epsilon_t$, where u_t denotes the VAR reduced-form residuals, knowledge about the structural impact multipliers in A_0^{-1} is sufficient for recovering the structural objects of interest. To avoid overfitting, we use Bayesian estimation techniques, imposing Minnesota-style Normal-Wishart priors as in Kadiyala and Karlsson (1997).[7] The overall tightness is set to 0.2, and the degrees of freedom parameter is set to $n + 2$.

2.2 Data

We estimate the reduced-form representation of the model outlined above using monthly time series for Germany covering 1999:1–2022:12. Data on domestic natural gas supply, the cross-border price, and inventories are obtained from the Federal Office for Economic Affairs and Export Control (BAFA). We define supply as the sum of imports and (a small

role of inflation expectations for the propagation of natural gas price shocks in the euro area.

[6] We use eleven instead of twelve seasonal dummies to avoid singularity of the regression matrix due to the presence of the intercept vector c (Kilian and Lütkepohl, 2017).

[7] Using an uninformative prior leads to qualitatively identical but somewhat more erratic results.

amount of) domestically produced natural gas less exports. As a measure of real domestic activity, we use the German industrial production (IP) index excluding construction activity. The real gas price corresponds to the cross-border price of gas deflated by the German consumer price index (CPI). Following Baumeister and Hamilton (2019), changes in natural gas inventories enter the model as a fraction of the previous month's gas supply. Natural gas supply and the real gas price enter the model in percentage growth rates, while the seasonally unadjusted industrial production index enters in logs.[8] Finally, we obtain the average monthly temperature from the German Weather Service (DWD).

2.3 Identification

Our goal is to set-identify three structural disturbances: flow supply shocks, aggregate demand (sometimes called flow demand) shocks, and storage demand (sometimes called speculative demand) shocks. We leave one shock unrestricted, which encompasses other drivers of the demand for natural gas, such as shifts in preferences for or advancements in storage technology. Each shock is normalized such that it raises the real price of natural gas. We achieve set-identification by imposing sign restrictions on the impulse response functions of the endogenous variables along the lines of Kilian and Murphy (2014). These restrictions are summarized in Table 1.

Following the oil market literature, a flow supply shock is assumed to move German gas supply and economic activity in the same direction, whereas the gas price moves in the opposite direction. An aggregate demand shock instead moves German gas supply, economic activity, and the real gas price in the same direction. As stressed by Kilian and Murphy (2014), the response of inventories is ex ante ambiguous. On the one hand, both a negative flow supply and a positive aggregate demand shock may cause a reduction of gas inventories. On the other hand, the anticipation that either of these shocks raises natural gas prices in the future may increase the demand for inventories already today. Accordingly, we abstain from restricting the impact response of inventories.

In response to a storage demand shock, gas supply, real gas price, and gas inventories move in the same direction, whereas economic activity moves in the opposite direction. This guarantees that an exogenous increase in storage demand is not conflated with an exogenous disruption of gas supply (i.e. a negative flow supply shock) or an exogenous reduction of economic activity (i.e. a negative aggregate demand shock).

While each admissible model satisfies by construction the sign restrictions in Table 1, not all set-identified models are equally plausible from an economic perspective. To narrow down the set of admissible models, we impose a small number of so-called narrative sign restrictions (NSR), as proposed by Kilian and Murphy (2014) and formalized by

[8] Figure A.2 in Appendix A.1 plots the time series of the endogenous variables, as they enter the SVAR model in equation (1).

Table 1: Sign restrictions on impact responses

	Flow supply shock	**Aggregate demand shock**	**Storage demand shock**
Gas supply growth	−	+	+
Industrial production	−	+	−
Real gas price growth	+	+	+
Gas inventories			+

Notes: + and − indicates a positive and negative response, respectively. Missing entries mean that no sign restriction is imposed. All sign restrictions are imposed as weak inequality constraints on impulse response functions in the period of the shock.

Antolín-Díaz and Rubio-Ramírez (2018). The key idea is to select the economically most plausible candidate models by restricting the sign of a given structural shock or its contribution to the historical decomposition of the endogenous variables during selected episodes in line with a widely accepted narrative. In addition to the sign restrictions in Table 1, we therefore require that admissible models must satisfy the following NSRs:

1. The gas supply shock was *negative* in January 2009, when the Russia-Ukraine transit dispute led to an unexpected halt of natural gas flows through Transgas between January 7 and January 20 (see, e.g., Nick and Thoenes, 2014).

2. The gas supply shock was *negative* in June and July 2022, when Russia unexpectedly reduced the natural gas flow through Nord Stream 1 to 50% and zero, respectively (see Figure A.3 in Appendix A.1).

3. For the periods specified by Restriction 2, gas supply shocks are the *overwhelming* contributor to unexpected movements in gas supply.

4. The aggregate demand shock is the *overwhelming* contributor to the unexpected reduction in industrial production at the start of the COVID-19 pandemic in April 2020 (see, e.g., Balleer, Link, Menkhoff, and Zorn, 2022).

We obtain model draws satisfying both sign and narrative restrictions using the rejection sampler of Rubio-Ramírez, Waggoner, and Zha (2010) as well as the importance sampler of Antolín-Díaz and Rubio-Ramírez (2018).[9]

[9] As stressed by Baumeister and Hamilton (2019), the prior on the orthogonal rotation matrix, which is commonly imposed in SVARs with sign restrictions (see, e.g., Uhlig, 2005; Rubio-Ramírez et al., 2010; Arias, Rubio-Ramírez, and Waggoner, 2018), may be unintentionally informative. Whether this concern is empirically relevant is an ongoing debate, though. Inoue and Kilian (2021) show that, in models with multiple sign restrictions and further restrictions, such as narrative restriction, the impact of the prior tends to be small. Moreover, resorting to the approach of Baumeister and Hamilton (2019) requires a-priori beliefs about the structural coefficients. Given that the existing literature does not warrant forming such priors for the (German) natural gas market, and our model contains several identifying restrictions, we follow the standard approach of Rubio-Ramírez et al. (2010).

2.4 Importance of narrative restrictions

To assess the relevance of the narrative restrictions, we follow Antolín-Díaz and Rubio-Ramírez (2018) and calculate rejection rates both individually and jointly (see Table 2). It is important to note that a high rejection rate should not be interpreted as evidence against the plausibility of a particular NSR. Instead, it suggests that the baseline specification encompasses structural parameters that are at odds with the narrative evidence.

The restrictions on the signs of gas supply shocks in January 2009 (NSR 1) and mid-2022 (NSR 2) are mildly informative. About 10% of the models identified based on conventional sign restrictions do not satisfy them individually, while 20% do not satisfy them jointly, suggesting that both restrictions add unique information. The restrictions on the contribution to the historical decomposition in April 2020 (NSR 3) and mid-2022 (NSR 4) have substantially more bite. About 50% and 60% of the candidate models do not satisfy NSR 3 and NSR 4, respectively. Imposing them jointly further shrinks the set of admissible models to less than 20% of the models identified by conventional sign restrictions alone, suggesting again that both NSRs carry relevant information. Interestingly, the restrictions on the historical decompositions are consistent with those on the signs of shocks, as adding the latter leaves the rejection rate virtually unchanged. In total, the results suggest that the narrative restrictions add valuable information to the identification process and eliminate models that entail implausible structural estimates.

Table 2: Rejection rates for narrative restrictions

Restrictions	**NSR 1**	**NSR 2 & 3**	**NSR 4**	**NSRs in row**
Signs of shocks	10.8	11.9	–	18.9
Historical decompositions	–	49.3	60.5	80.3
Joint restrictions	10.8	50.1	60.5	81.5

Notes: Rejection rates in % of models identified based on sign restrictions for each narrative restriction (NSR) imposed individually and for (sub-)sets of NSRs imposed jointly.

3 Baseline Results

In this section, we investigate the average effects of the identified structural shocks. We start by discussing their long-run contributions to the forecast error variance decomposition. We then analyze the impulse responses to each of the structural shocks. Finally, we quantify their contributions to the historical decompositions of the endogenous variables.

3.1 What drives dynamics in the German gas market?

As a starting point, we assess the contribution of each structural shock to the unconditional variance of the endogenous variables on average over the sample period. Table 3

Table 3: Contribution of structural shocks to FEVD (in %)

	Flow supply shock	**Aggregate demand shock**	**Storage demand shock**
Gas supply growth	29.2	10.9	41.0
Industrial production	10.3	57.1	17.5
Real gas price growth	53.4	25.0	9.7
Gas inventories	12.5	12.4	33.0

Notes: Posterior means of forecast error variance decomposition (FEVD) based on models satisfying both conventional and narrative sign restrictions. Unconditional variances are approximated by setting the forecast horizon to $h = 100$ months. Residual contributions to the FEVD accounted for by other gas demand shocks.

reports posterior means of the forecast error variance decomposition (FEVD) after 100 periods, which approximates the unconditional variance. Flow supply, aggregate demand, and storage demand shocks account for the vast majority of the FEVD for each variable, amounting to between 80 and 90% for gas supply growth, industrial production, and real gas price growth and about 60% for gas inventories.

Note that 41% of the unconditional variance of gas supply growth is explained by storage demand shocks, suggesting that gas inventories are used for speculative trading or that German storage capacity might be too low, requiring frequent sizeable changes in natural gas supply. Another third of the variance is accounted for by flow supply shocks, while aggregate demand shocks only contribute about 11% to the FEVD of gas supply growth. The latter finding may be rationalized by the fact that natural gas has become an important energy carrier in the recent past. Consistently, flow supply shocks explain a mere 10% of the unconditional variance of industrial production, whereas the majority is attributed to aggregate demand shocks. In light of German industry's strong export dependency, this appears plausible.

Gas supply and demand shocks account for 90% of the unconditional variance of real gas price growth, with the largest fraction attributed to flow supply shocks. Close to one third of unexpected fluctuations in gas inventories is due to storage demand shocks, whereas flow supply and aggregate demand shocks together merely account for another quarter. Accordingly, the remaining 42% of the unconditional variance of gas inventories is explained by the residual shock, which does not have a clear economic interpretation. One candidate explanation for the latter findings are non-economic factors, such as geo-strategic or political considerations.[10]

[10]During 2021, for example, Gazprom Germania neglected refilling its gas storage facilities in Germany, leading to exceptionally low levels of gas inventories in March 2022 (see Figure A.1 in Appendix A.1).

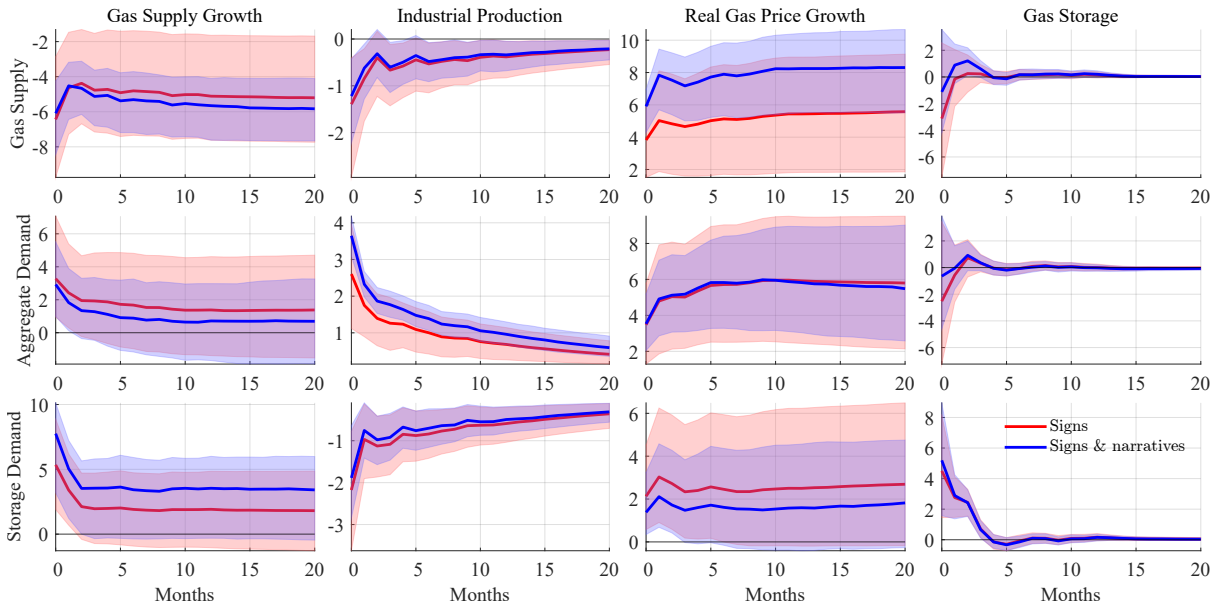

Figure 2: Impulse response functions to structural gas supply and demand shocks

Note: Red lines and shaded areas indicate point-wise median IRFs and 68% equal-tailed posterior density intervals based on the SVAR identified by conventional sign restrictions. Blue lines and shaded areas indicate the corresponding objects based on the SVAR identified by conventional and narrative sign restrictions.

3.2 Impulse response analysis

Figure 2 plots point-wise medians and 68% equal-tailed posterior density intervals of the impulse response functions (IRFs) for the SVAR identified by conventional sign restrictions alone (in red) and the SVAR model identified by conventional and narrative sign restrictions (in blue). The IRFs of gas supply and real gas price growth are accumulated and are reported in levels.[11]

First, consider the IRFs to a flow supply shock in the top row. According to either identification scheme, a negative flow supply shock leads to persistently lower gas supply and a persistently higher real gas price. The increase in gas prices leads to a reduction of industrial production (IP). While the largest drop lasts for only two months, IP remains slightly below steady state even two years after the shock. Gas inventories, which were left unrestricted, increase for about four months. While the IRFs are qualitatively robust to the identification scheme, the SVAR identified by conventional and narrative sign restrictions suggests that a flow supply shock of similar magnitude induces a larger increase in the real gas price, albeit an almost identical reduction of IP. Intuitively, the narrative sign restrictions are consistent with model draws that imply a smaller elasticity between gas prices and economic activity. For example, the gas supply cuts in June and July 2022 captured by NSR 2 were followed by large price increases but moderate reactions of German IP.

[11]The IRFs of changes in gas inventories, which are expressed in percent of the previous period's gas supply, are not cumulated.

A positive aggregate demand shock leads to a persistently higher economic activity, which remains significantly above steady state for at least two years after the shock. The increase in IP goes along with a persistent increase of gas supply. The real price of natural gas also increases for about ten months before starting a slow decline. Gas inventories do not respond strongly to the aggregate demand shock. With narrative restrictions, the impact on both gas supply and IP is somewhat larger on impact, while the real gas price exhibits an almost identical response. Gas inventories fall on impact and slightly overshoot after a few months.

A positive storage demand shock induces persistently higher levels of gas supply and the real gas price. Economic activity drops strongly in the first two months and remains slightly below steady state for more than a year. The dynamics of IP are similar to those following a negative flow supply shock. However, gas supply drops while it increases in response to the storage demand shock, and the gas price increases considerably more in the supply-induced case. Nevertheless, the storage demand shock, in particular, exerts strong negative effects on real economic activity. The response of changes in gas storage growth is hardly affected by the addition of narrative restrictions.

From Figure 2, imposing narrative restrictions primarily affects the IRF of the real gas price to each of the structural shocks. Without narrative restrictions, its median response to a negative flow supply and a positive aggregate demand shock is of the same order of magnitude. With narrative restrictions, the price response to a negative flow supply shock is about three times as large. In light of the recent turmoil in the German gas market, the latter result appears much more plausible.

3.3 Historical decomposition

The contributions to the long-run FEVD in Table 3 indicate that gas supply and demand shocks are important drivers of fluctuations in the German natural gas market. However, they are mute about their importance during selected episodes. Beyond their contribution on average over the sample period, we are interested in how each shock contributed to fluctuations of the endogenous variables over time and, in particular, during the recent turmoil in the German gas market. In a first step, we consider the period 2000:1–2018:12. We then zoom in on the energy crisis at the end of our sample period.

Figure 3 plots point-wise median estimates of the cumulative effect of each structural shock on real gas price growth, which is depicted by the dashed lines. Note, in particular, that the residual shock contributes very little to fluctuations in real gas price growth, confirming that the model explains its dynamics based on the identified structural shocks. The historical decomposition suggests that, at the start of the sample, fluctuations in real gas price growth were almost exclusively driven by flow supply shocks, while all three structural shocks contributed in the aftermath of the global financial crisis. Towards the

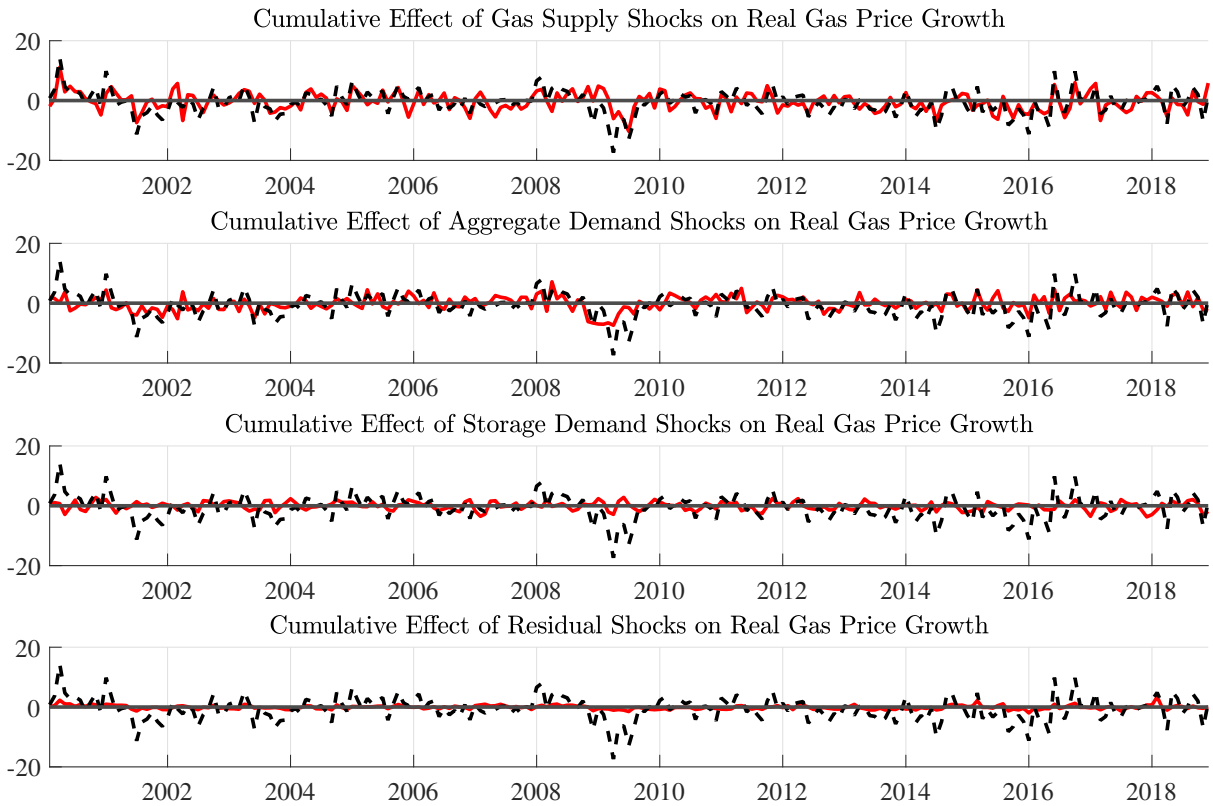

Figure 3: Historical decomposition of German real gas price growth for 2000:2–2018:12

Note: Dashed lines show deviations of actual data from the deterministic trend. Solid lines show the contribution of each shock to the deviation from the deterministic trend.

end of the sample, flow supply and storage demand shocks appear to be the main drivers of fluctuations in real gas prices.

Similarly, Figure 4 plots point-wise median estimates of the cumulative effect of each structural shock on gas supply growth. For the entire sample period, the general pattern of change of gas supply is dominated by flow supply shocks, whereas aggregate demand shocks play only a moderate and less systematic role. Consistent with their large contribution to the FEVD (see Table 3), storage demand shocks have been an important driver of fluctuations in German gas supply, although we control for both seasonal patterns and average monthly temperature. Especially during the early 2000s and in 2017–2018, gas supply growth seems to be strongly affected by storage demand shocks.[12]

In the aftermath of the Russian invasion of Ukraine, Germany experienced a dramatic surge of energy prices (see Figure 1). In what follows, we assess the structural drivers of fluctuations in the German gas market during this episode. Accordingly, Figure 5 depicts the historical decomposition of the endogenous variables based on the SVAR identified

[12]The decompositions of the other two variables are provided in Appendix A.1. Figure A.4 illustrates that German IP was predominantly affected by aggregate demand shocks in the first half of the sample, whereas storage demand shocks contributed to a similar extent in the second half. Figure A.5 shows that, in contrast to the other endogenous variables, the residual shock has some explanatory power for changes in natural gas inventories, in particular towards the end of the sample.

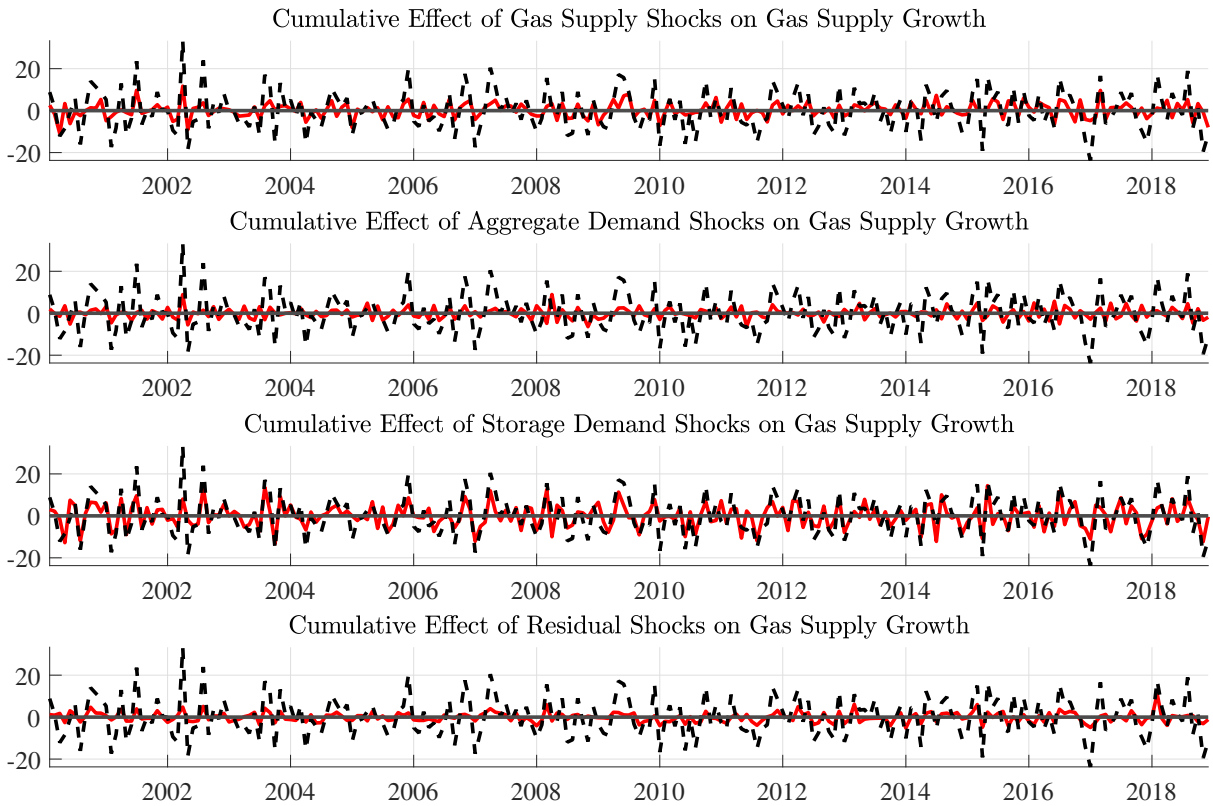

Figure 4: Historical decomposition of German gas supply growth for 2000:2–2018:12

Note: Dashed lines show deviations of actual data from the deterministic trend. Solid lines show the contribution of each shock to the deviation from the deterministic trend.

by conventional and narrative sign restrictions for 2020:1–2022:12.

In the top left panel, gas supply growth fluctuated around zero with no obvious trend from early 2020 until early 2022, when Russia started to reduce gas flows to Germany. From April 2022 onward, gas supply growth remained below its deterministic trend for several months, mainly due to the negative effects of flow supply and storage demand shocks. In September and October of 2022, the cumulative effect of flow supply shocks reversed, as the disruption of Russian gas supply was offset by higher imports especially from Norway (see Figure A.3 in Appendix A.1).

In the bottom left panel, negative flow supply shocks contributed to strong gas price growth during most months in 2020–2022. It is important to recall that NSR 3 only requires that flow supply shocks are the dominant driver of real gas price growth in June and July 2022, whereas their signs and contributions are unrestricted for the rest of the time period in Figure 5. Nevertheless, flow supply shocks clearly dominate fluctuations in real gas prices throughout the entire episode. In July 2022, gas price growth would have been 30% lower without the cumulative effect of negative flow supply shocks. In October 2022, when an increase in supply from other countries partially offset the lack of imports from Russia, gas price growth would have been 30% higher without the cumulative effect of positive flow supply shocks. In June–August 2022, positive storage demand shocks also

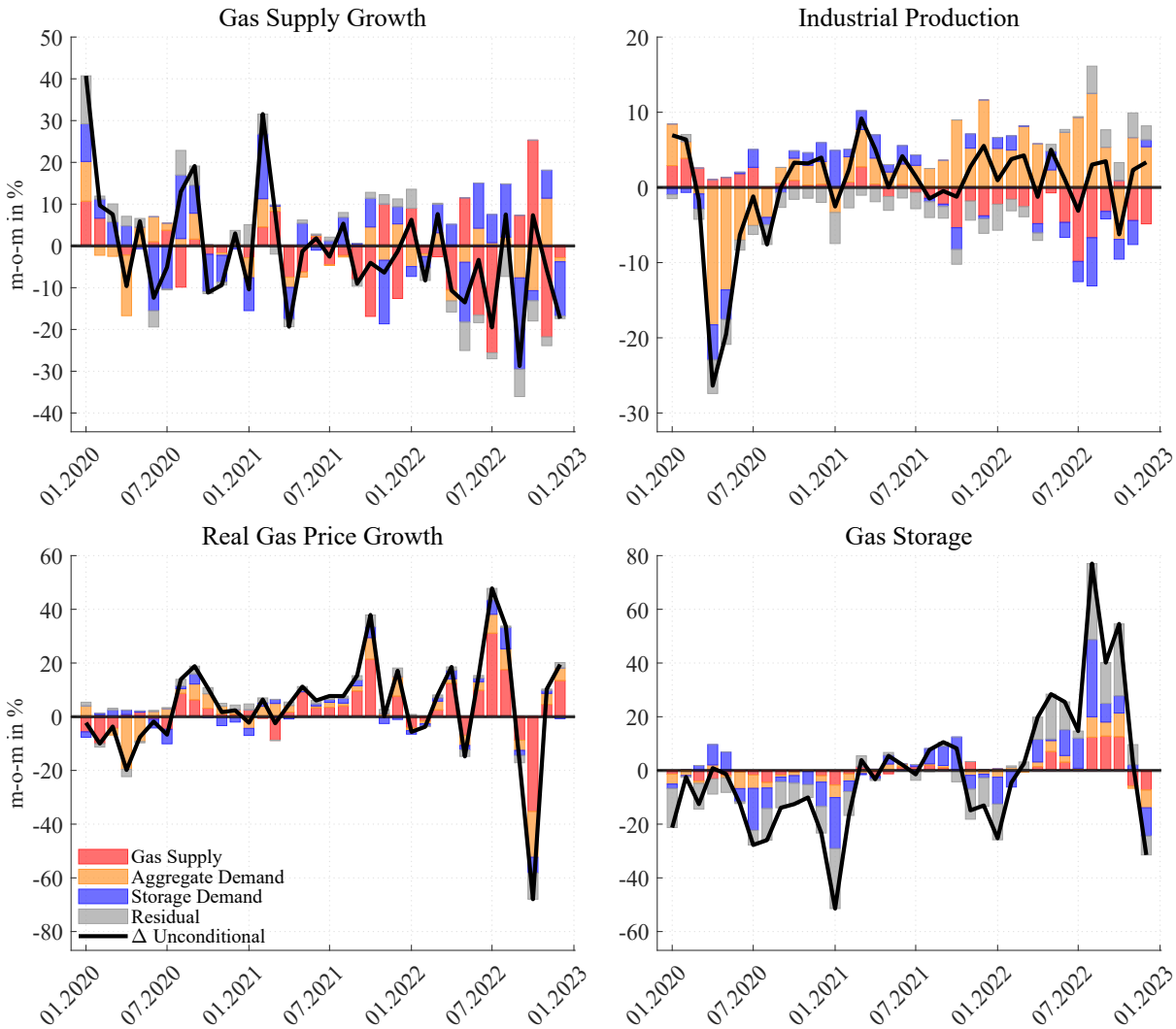

Figure 5: Historical decomposition for 2020–2022.

Note: The black line is the deviation of the respective series from its unconditional forecast. The bars depict the contribution of the shocks to this deviation.

exerted substantial upward pressure on real gas price growth, while a substantial part of the relaxation in October 2022 is attributed to negative aggregate demand shocks. The latter effects are consistent with the political decision to ramp up gas inventories prior to the winter and successful measures to save on natural gas use by the German industry in fall 2022, respectively.

The successful efforts of the German industry are also reflected by the robust development of IP in the top-right panel. After a large drop in economic activity during the COVID-19 pandemic, which is largely attributed to negative aggregate demand shocks, IP swiftly recovered mostly due to positive aggregate demand shocks.[13] From mid-2021 onward, increasingly negative effects of flow supply shocks offset the positive effects of

[13]Strong aggregate demand in the aftermath of the pandemic arguably reflects catch-up effects after the relaxation of lock-down measures and supply-chain frictions as well as fiscal support measures (see, e.g., Bachmann et al., 2021; Balleer et al., 2022).

aggregate demand shocks and slowed down the recovery of the German industry. In 2022, positive storage demand shocks exerted further downward pressure on IP, consistent with the German parliament's decision to ramp up gas inventories before the upcoming winter. Until the end of our sample period, the combined negative effects of flow supply and storage demand shocks were roughly offset by the positive cumulative effects of aggregate demand shocks, and German IP remained close to its deterministic trend. The robustness of IP during this energy crisis arguably reflects the successful attempts of the German industry to reduce its dependence on natural gas in the face of political requirements following the Russian invasion of Ukraine.[14]

While all structural shocks contributed to the draw-down of natural gas inventories in 2020, the SVAR model attributes the latter to a few negative aggregate demand shocks and a series of negative storage demand shocks. In light of the drop in energy prices at the start of the COVID-19 pandemic, economic agents arguably speculated on an extended period of lower natural gas prices and therefore reduced inventory holding.

The historical decomposition in Figure 5 provides both a plausibility check of our SVAR model during an important episode, where we can draw on narrative evidence, and a quantitative analysis of the recent turmoil in the German natural gas market. Our results suggest that, while the economy was hit by severe gas supply and demand shocks leading to a dramatic price hike, the consequences for real economic activity were rather modest, not unlike the theoretically founded predictions in Bachmann et al. (2022).

4 Structural Scenario Analysis

The benefit of our framework is that we can conduct structural scenario analyses along the lines of Antolín-Díaz, Petrella, and Rubio-Ramírez (2021) by assuming hypothetical realizations for one or more of the structural shocks and investigating the resulting time paths of the endogenous variables. Subsequently, we consider the counterfactual scenario of an embargo on gas imports from Russia starting in April 2022 and the importance of a milder winter 2022/2023 for German gas inventories.

4.1 Russian gas embargo

In response to the Russian invasion of Ukraine on February 24, 2022, the U.S. government and the European Council jointly with other governing bodies have adopted a number of restrictive measures to weaken Russia's economic base.[15] As part of the "Fifth package of sanctions in response to Russia's invasion of Ukraine", the European Council banned

[14]Relative to the 2018–2021 average, German industrial natural gas consumption was about 15% and 18% lower in 2022 and 2023, respectively (see Figure A.6 in Appendix A.1).

[15]The EU has also adopted sanctions against Belarus and Iran for their involvement in the invasion of Ukraine and the supply of drones, respectively (see European Council).

imports of coal and other fossil fuels from Russia on April 8, 2022. A ban on oil imports from Russia was discussed and implemented in the form of a price cap at $60 per barrel for crude oil and petroleum oils by the EU and the G7 member states on December 5, 2022. Although a ban on natural gas imports from Russia was also discussed by German economists (e.g. Bachmann et al., 2022; Krebs, 2022), politicians, and the media, it was not implemented before Russia itself throttled natural gas exports to Germany in June and July and eventually suspended them altogether in September 2022.

Suppose instead that Germany had taken initiative and banned natural gas imports from Russia as early as April 2022. A complete halt without immediate substitution would have reduced German gas imports by 49%. Assuming that gas exports also decreased by 49% and domestic production developed as actually observed in the data, German natural gas supply would have dropped by 46.2%. In the SVAR model, this corresponds to a *one-off six unit negative flow supply shock* in April 2022.[16] Subsequently, we assume parameter estimates for the full sample and realized monthly average temperatures for 2022:4–2023:3. Thus, we can compare conditional forecasts of the endogenous variables both to the unconditional forecasts and to the realized data (up to December 2022).

Figure 6 plots the 'Scenario' forecast with and the 'Baseline' forecast without a six unit negative flow supply shock in April 2022 against the data for each of the endogenous variables. As intended, the negative gas supply shock leads to a substantial drop in the monthly growth rate of gas supply relative to the baseline in April, followed by a slight overshooting in May (top left panel). From June onward, both growth rates move closely together. The actual data, in turn, reflect the gradual reduction of imports from Russia in April through July, the temporary reactivation of Nord Stream 1 and Transgas in August, and the final suspension of exports to Germany in September 2022 leading to deviations from both the embargo scenario and the baseline forecast.

Consistently, the scenario forecast implies substantial upward pressure on gas price growth relative to the baseline forecast in the period of the shock (bottom left panel). In the data, a similar price increase is visible, but three months later than in the embargo scenario forecast. According to the historical decomposition in Figure 5, this reflects the unfortunate combination of negative supply shocks due to the Russian cuts of gas exports to Germany and positive storage demand shocks due to the political decision to ramp up German gas inventories prior to the start of the heating season.

In the embargo scenario forecast, industrial production (top right panel) would have dropped in April 2022 compared to the baseline forecast, although the latter is included in the 68% posterior credibility set of the former. In the following months, the paths of IP in the embargo scenario and the baseline forecast are similar, and the actual data is inside the 68% posterior credibility set of the scenario forecast. This reflects our earlier finding

[16] Month-on-month reductions in German natural gas supply of 20% or more occur on at least ten occasions during our sample period (see Figure A.2 in Appendix A.1).

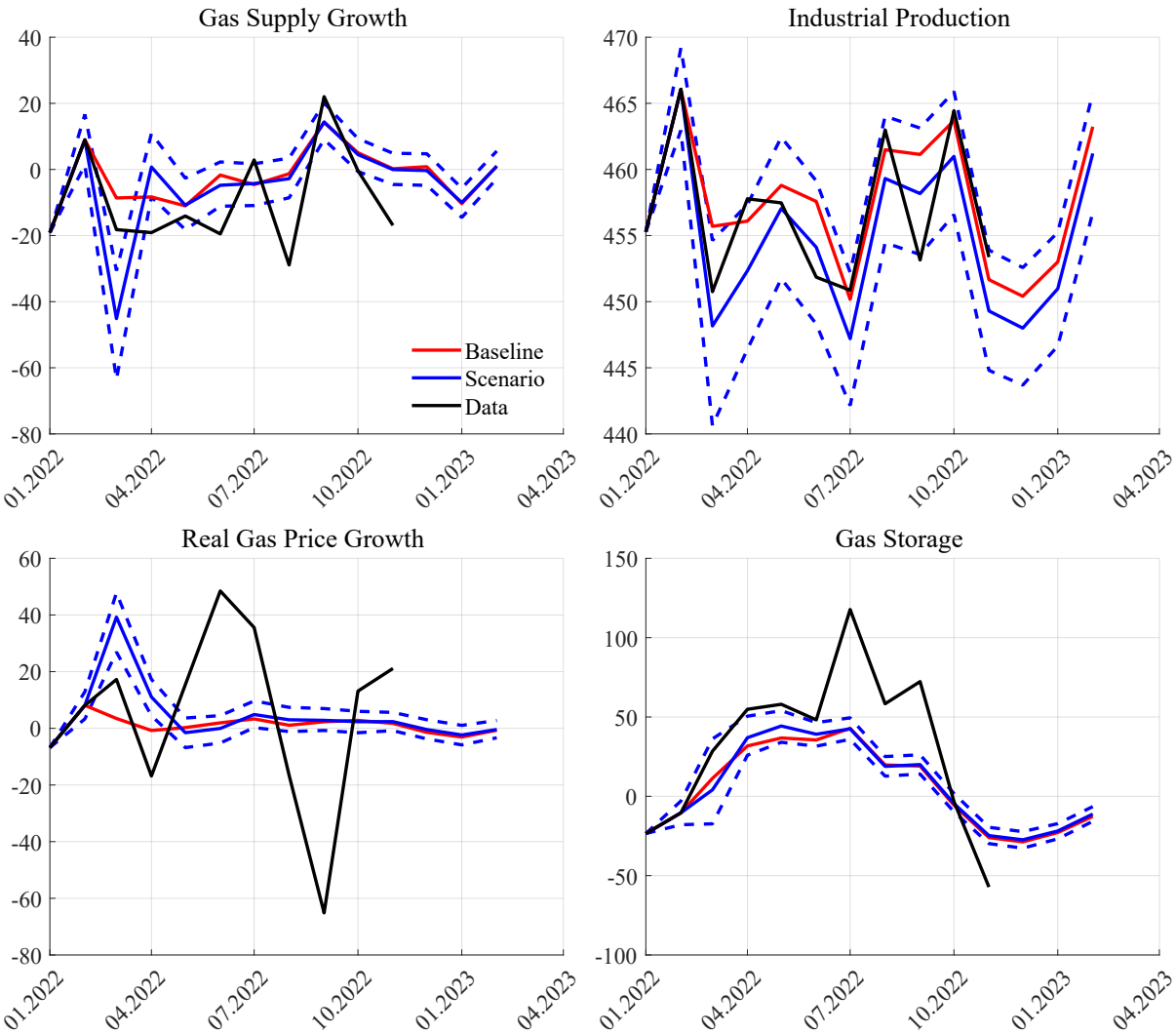

Figure 6: Conditional forecasts for a Russian gas embargo starting in April 2022

Note: Point-wise median conditional forecasts with 68% posterior credibility sets based on the SVAR identified by conventional and narrative sign restrictions

that supply effects on economic activity are short-lived and moderate in comparison with price effects, as illustrated by the impulse response functions in Figure 2.

Both the embargo scenario and the baseline forecast track actual changes in natural gas inventories (bottom right panel) up to July 2022. Starting in August, gas inventories increased strongly due to the concerted attempt to fill underground storages (see Figure A.1) leading to strong deviations from the embargo scenario and the baseline forecast.

4.2 The role of temperature

In the media as well as the reports of economic and policy institutions (see, e.g., Joint Economic Forecast, 2022; Deutsche Bundesbank, 2022), the risk of a natural gas shortage in Europe and Germany, in particular, was repeatedly linked to the severity of the winter 2022/2023. The SVAR model in (1) accounts for seasonal variation in the form of monthly

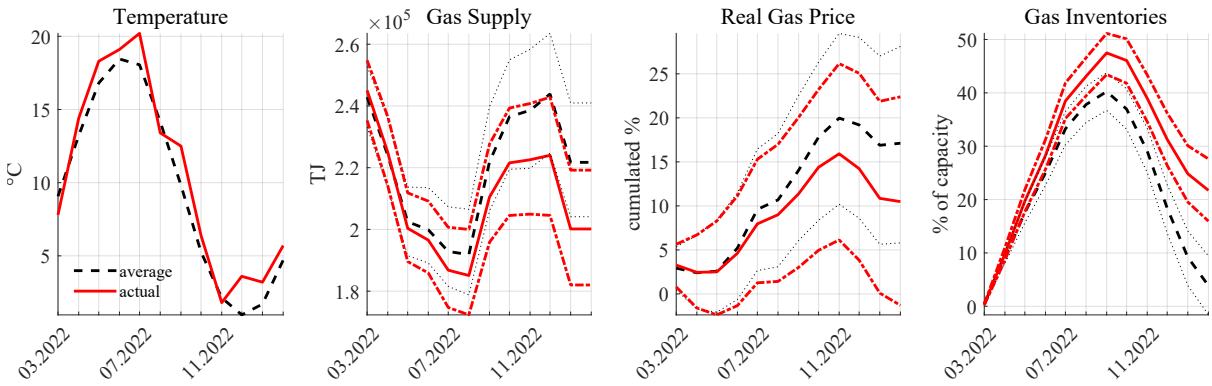

Figure 7: Conditional forecasts for average and actual monthly temperatures for April 2022 through March 2023

Note: Point-wise median conditional forecasts with 68% posterior credibility sets based on the SVAR identified by conventional and narrative sign restrictions

dummies as well as average monthly temperatures. We can therefore investigate whether different temperature scenarios indeed imply substantially different time paths of the endogenous variables.

Figure 7 plots the actual monthly temperatures for April 2022 through March 2023 against sample averages as well as point-wise median forecasts and 68% posterior credible sets of natural gas market variables based on the model with narrative sign restrictions conditional on these temperature paths. To facilitate interpretation, the effect on gas supply and gas price is cumulated to levels in terra joules (TJ) and percent, respectively, while the effect on gas inventories is converted to a fraction of German capacity.[17]

From the first panel of Figure 7, temperatures were consistently higher in April 2022 through March 2023 than on average during our sample period. The second panel shows that higher average monthly temperatures implied a cumulated reduction of gas supply by 2,000 TJ during the same period. Higher temperatures also resulted in up to 5 percentage points lower gas prices in the second half of the forecast horizon. Most importantly, we find that gas inventories would have been about 18% (of full capacity) lower in a year with average monthly temperatures. Given that German gas inventories were down to 25% of full capacity in March 2022 (see Figure A.1), differences in average monthly temperature indeed seem to account for a non-trivial part of the variation in inventories. This finding lends ex-post empirical support to the political decision in April 2022 to ramp up German gas inventories before the start of the winter.

While the temperature-dependent conditional forecasts of gas supply and real gas price are quantitatively, albeit not statistically different, the last panel of Figure 7 indicates that

[17]Note that gas inventories enter the model as a percentage of the previous month's gas supply. To convert this into percent of capacity, we compute the conditional forecast of gas supply in TJ over the forecast horizon. We then compute the conditional forecast of gas inventory changes in TJ and cumulate over the forecast horizon. We finally convert TJ to MWh (1 TJ = 277.778 MWh) and divide by the total capacity of 230 million MWh of the 47 gas storage facilities located in Germany (see ENBW, 2020).

natural gas inventories were also statistically higher due to milder-than-average monthly temperatures between April 2022 and March 2023.

4.3 Disruption of Europipe I and II

As of 2023, Germany is relying on natural gas imports from Belgium, the Netherlands, and Norway as well as a slowly increasing capacity of LNG import terminals. Europipe I and II, which transport gas from Norway to Germany, accounted for 19.8% and 21.7%, respectively, of natural gas imports on average between July and December 2022.[18] Along the German coast, however, Europipe I and II run next to each other in shallow water, making them susceptible to targeted disruptions.[19]

For the same export and production assumptions as in the Russian gas embargo scenario, a disruption of natural gas imports from Norway corresponds to a drop in German gas supply by 17.6% (Europipe I), 19.3% (Europipe 2), or 36.9% (both). Accordingly, we analyze a structural scenario, in which we subject the SVAR model to a *one-off four unit negative flow supply shock*. We simulate this shock in January 2023 to trace out the effects over a full year, but any other starting point would lead to similar effects.

Figure 8 plots point-wise median conditional forecasts with 68% posterior credible sets of German gas supply, industrial production, gas import price, and gas inventories using the same transformations as in the temperature scenario without (dashed lines) and with (solid lines) the shock. The left panel illustrates the immediate drop in the level of gas supply due to the hypothetical disruptions of imports from Norway, which leads to an immediate increase in the real gas price by about 25% relative to the baseline, which persists over the rest of the forecast horizon. Nevertheless, the negative gas supply shock has only a moderate effect on IP on impact, which largely disappears before the end of the forecast horizon. In the first few months, gas inventories are actually 2% (of total capacity) higher than in the baseline, given that negative flow supply shocks tend to induce higher prices and saving efforts rather than a draw-down of inventories during our sample period (see Figure 2). During the second half of the forecast horizon, this relation reverses, albeit only temporarily.

In contrast to the temperature scenario in Figure 7, the conditional forecasts are statistically different for gas supply and, in particular, real gas price, whereas the posterior credibility sets for gas inventories overlap for much of the forecast horizon. Especially the latter finding of small or even positive effects on inventories must be taken with a grain of salt. Despite a number of non-trivial gas supply disruptions during our sample period, Germany was generally able to draw on alternative sources of imports or reduce

[18] See Figure A.3 in Appendix A.1 for details.

[19] In May 2023, UK Defence Secretary Ben Wallace and Norwegian Defence Minister Bjørn Arild Gram signed a security partnership to increase cooperation on undersea capabilities and counter threats to undersea infrastructure (www.gov.uk), signaling increased political awareness of related risks.

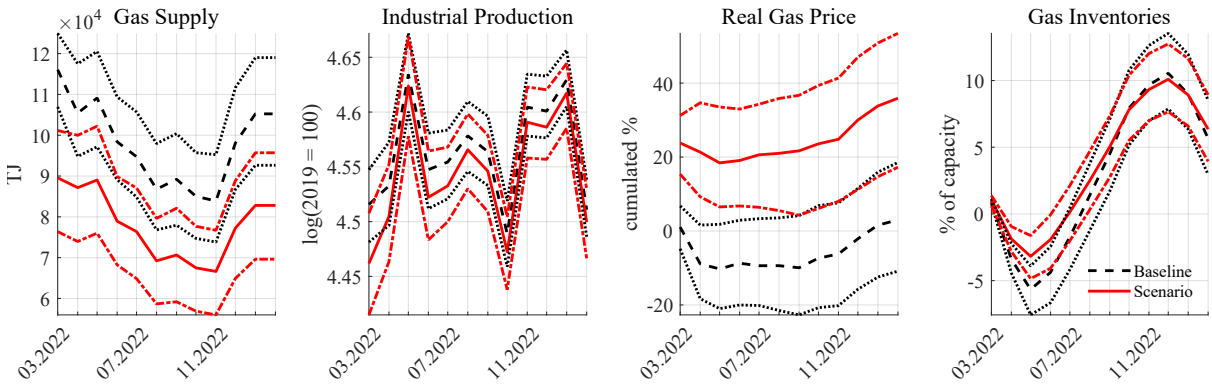

Figure 8: Conditional forecasts for a disruption of Europipe I and II in January 2023

Note: Point-wise median conditional forecasts with 68% posterior credibility sets based on the SVAR identified by conventional and narrative sign restrictions

its own exports. In the current situation, where Belgium, the Netherlands, and LNG are the only remaining outside options, it is unlikely that savings and substitution of natural gas will be equally smooth as in the past.

5 Robustness to Time-Varying Parameters

The recent turmoil in the German natural gas market raises the question whether a linear model is appropriate to capture the dynamics of the endogenous variables over the entire sample period. In particular, one might argue that larger shocks and higher prices than previously observed have affected the behavior of economic agents and thus the structural parameters of interest (a.k.a. the "Lucas critique"). In this case, our empirical results and structural scenario analyses based on a linear model estimated for the full sample would be inconsistent.

To allow for the possibility of parameter instability, we specify a time-varying parameter (TVP) version of our model with stochastic volatility (SV) and estimate it using the quasi-Bayesian approach of Petrova (2019). The latter is particularly suited for our purpose, given that it flexibly allows for both gradual and abrupt parameter change. The key idea of this approach is to derive a period-specific likelihood function by re-weighting each observation such that observations closer to the period under consideration receive a higher weight, whereas distant observations are down-weighted. The decay of weights is governed by a kernel and its bandwidth. We follow Petrova (2019) and Giraitis, Kapetanios, and Yates (2014), who use a normal kernel with bandwidth $\sqrt{T}$, where T denotes the sample size. This is asymptotically optimal (in terms of MSE minimizing) and results in assigning non-zero weights to about fifteen years of observations.[20]

Figure 9 plots the IRFs based on the TVP-SV-VAR model evaluated in May 2008 (black dotted lines), August 2014 (red dashed-dotted lines), and July 2022 (orange dashed

[20]For details, see Appendix A.2 or Petrova (2019).

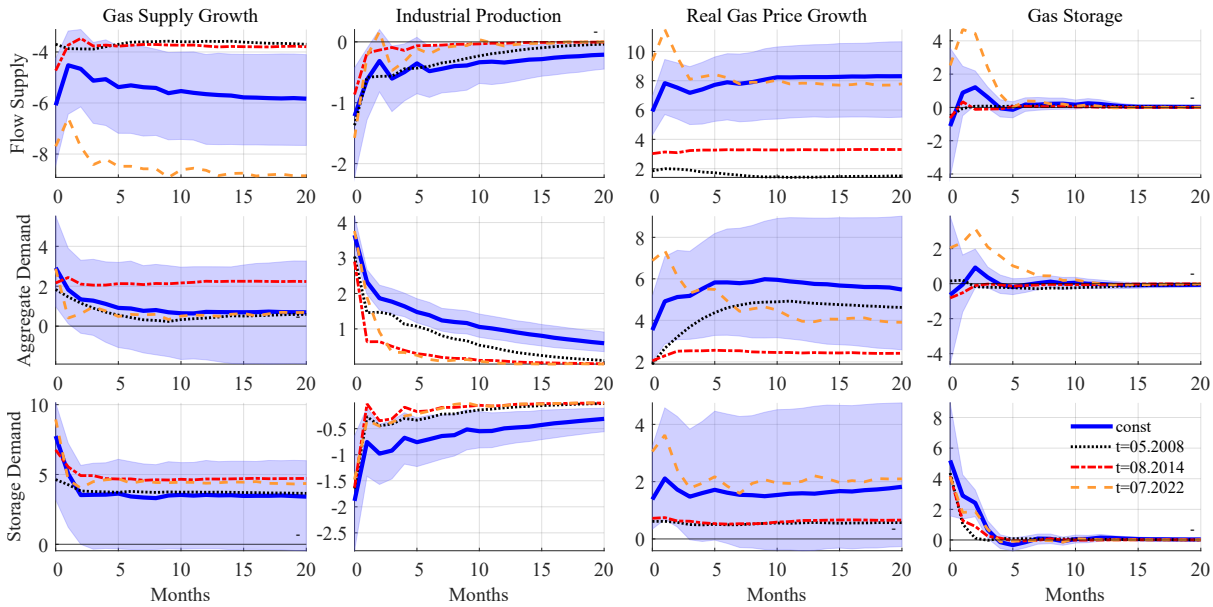

Figure 9: Impulse response functions based on constant-coefficient and TVP-SV-VAR models

Note: Solid lines and shaded areas indicate point-wise median IRFs and 68% equal-tailed posterior density intervals based on the constant-coeffient SVAR identified by conventional sign and narrative sign restrictions. Dotted, dashed, and dashed-dotted lines refer to point-wise median period-specific IRFs based on the TVP-SV-VAR model in Appendix A.2.

lines) against those for our baseline specification (blue solid lines and shaded areas). It is important to note that, for the first two sets of IRFs, observations associated with the energy crisis of 2022 receive a weight of zero, while we impose the same narrative sign restrictions as in the linear model.

Qualitatively, the IRFs from either model are rather similar, indicating that parameter instability is only a second-order concern. Quantitatively, the stronger flow supply shocks (i.t.o. of the drop in gas supply) estimated at the end of the sample, also induce a stronger response of gas prices and inventories, whereas the effects on industrial production are of similar magnitude. The impulse responses of gas supply, prices, and inventories to an aggregate demand shock seem to become more immediate and less persistent towards the end of the sample. Finally, storage demand shocks imply qualitatively and quantitatively similar responses of the endogenous variables in May 2008, August 2014, and July 2022.

With few exceptions, the IRFs based on the TVP-SV-VAR evaluated at different points in time fall within the point-wise 68% posterior density intervals based on the linear model estimated for the full sample. Accordingly, the empirical results in Section 3 and the scenario analyses in Section 4 are unlikely to be subject to the Lucas critique.

6 Conclusion

We propose a structural VAR model to disentangle the role of supply and demand shocks in the German natural gas market and conduct structural scenario analyses. The model

succeeds in explaining most of the variation in key variables based on the structurally identified shocks. Our estimates suggest that (i) supply and demand shocks have large and persistent price effects but moderate and short-lived output effects, (ii) the natural gas price hike of 2022 was largely driven by the Russian suspension of exports to Germany and the simultaneous attempt to ramp up gas inventories before the start of the winter, (iii) an immediate embargo on natural gas imports from Russia in April 2022 would have merely precipitated a price increase of similar magnitude, and (iv) a milder-than-average year helped substantially to maintain a robust inventory base throughout the last winter. Finally, we do not find strong evidence of time-varying parameters and thus different effects of natural gas supply and demand shocks at the end of our sample period.

In light of the recent disruptions of European natural gas markets, we build on SVAR tools developed for analyzing the global market for crude oil and apply state-of-the-art econometric methods to answer pressing questions. We thus provide empirical evidence that substantiates the political debate in similar situations. Given the backward-looking nature of our econometric approach, it should be clear that structural changes in the German gas market at the end of the sample period, such as the current expansion of LNG capacities or shifts in behavioral regularities and seasonal patterns, are incorporated only to a limited degree. Monitoring the role of these changes is left for future research.

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Appendix

A.1 Additional Figures

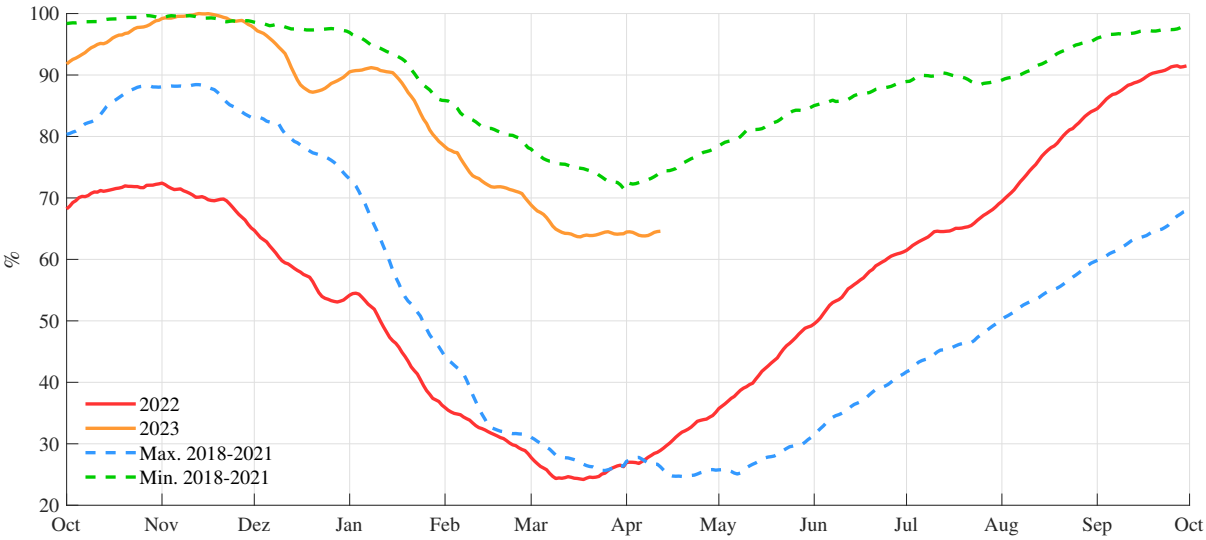

Figure A.1: Use of German natural gas storage capacity for October 2021 through April 2023

Source: Bundesnetzagentur

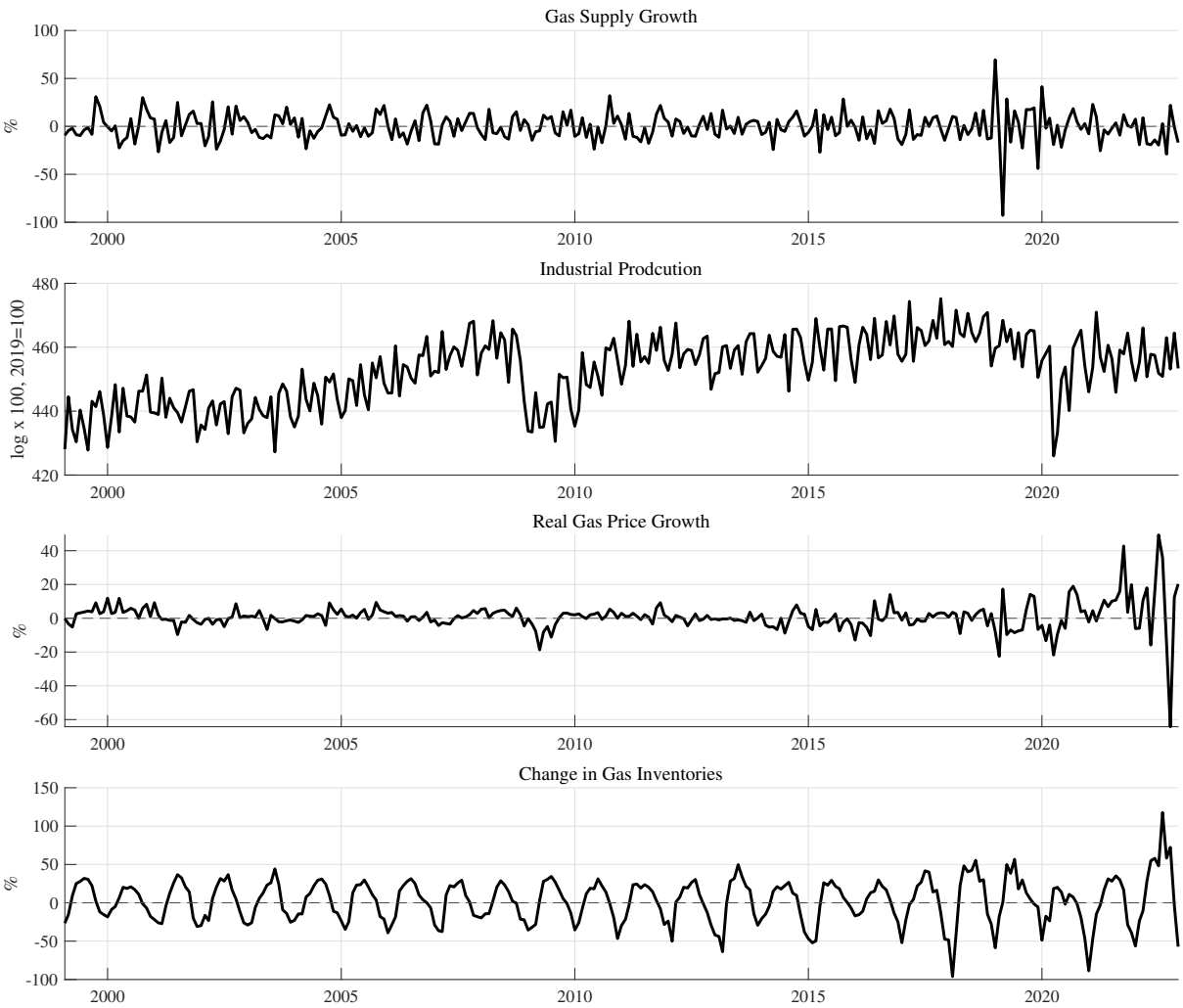

Figure A.2: Data on German natural gas supply growth, industrial production, real gas price growth, and gas inventories for 1999:2–2022:12

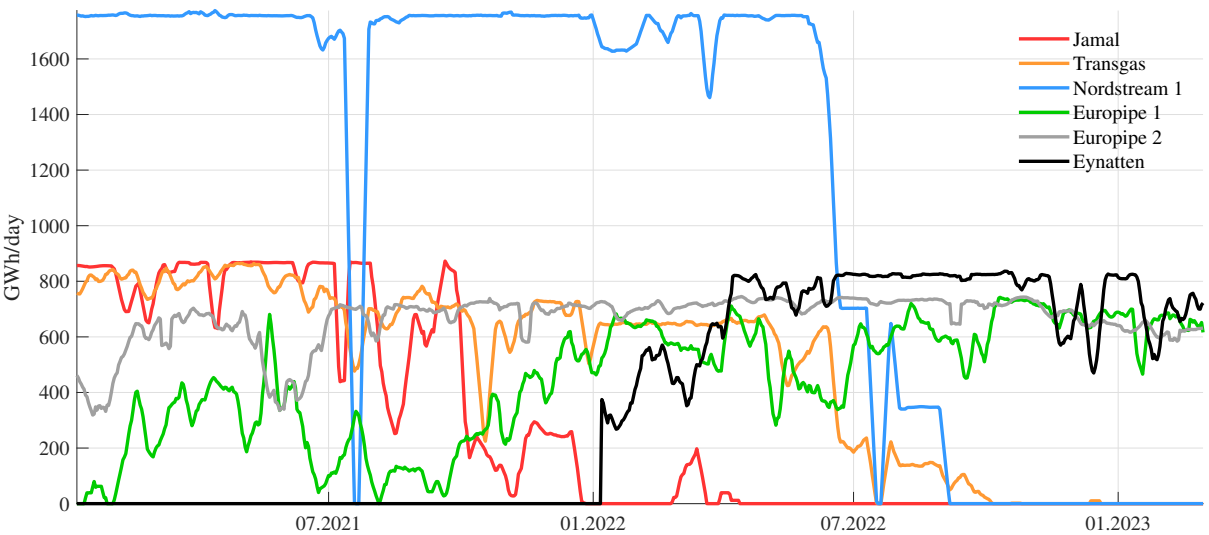

Figure A.3: Daily German natural gas import flows for selected pipelines

Sources: European Network of Transmission System Operators for Gas (ENTSO-G), German Statistical Office

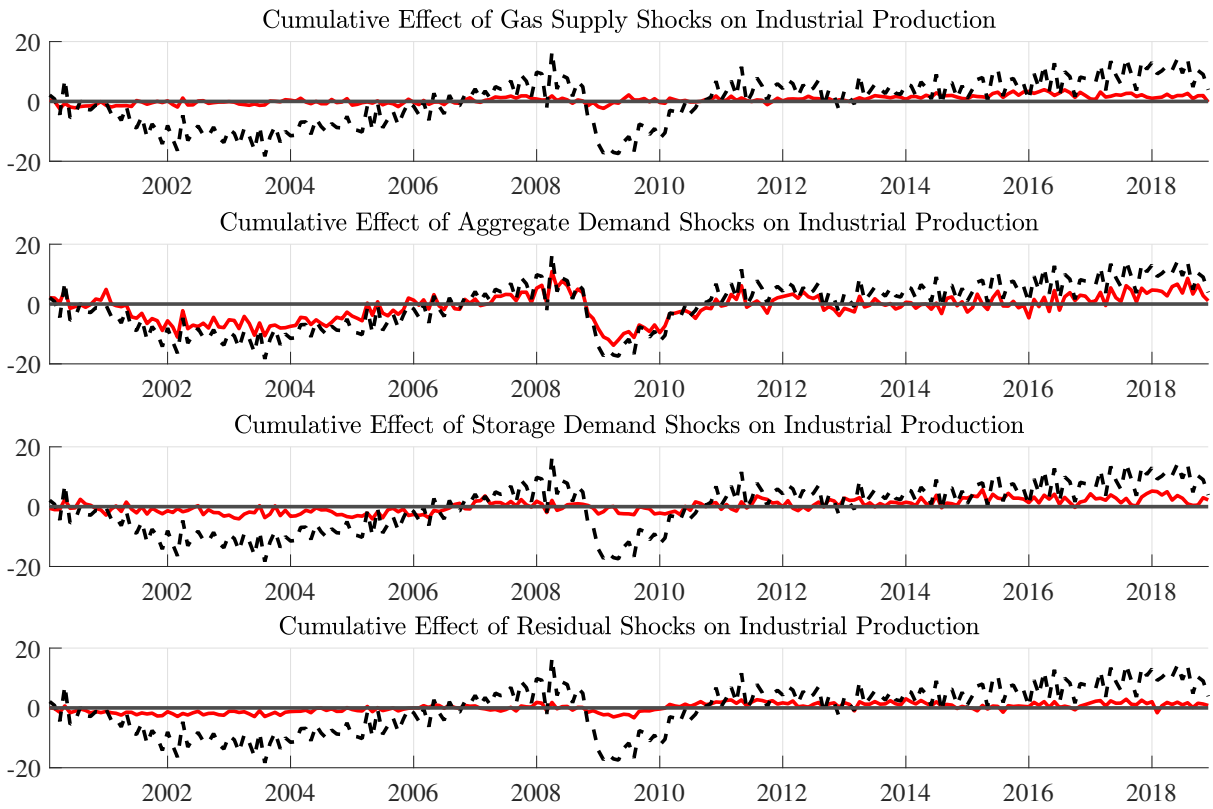

Figure A.4: Historical decomposition of German industrial production for 2000:2–2018:12

Note: Dashed lines show deviations of actual data from the deterministic trend. Solid lines show the contribution of each shock to the deviation from the deterministic trend.

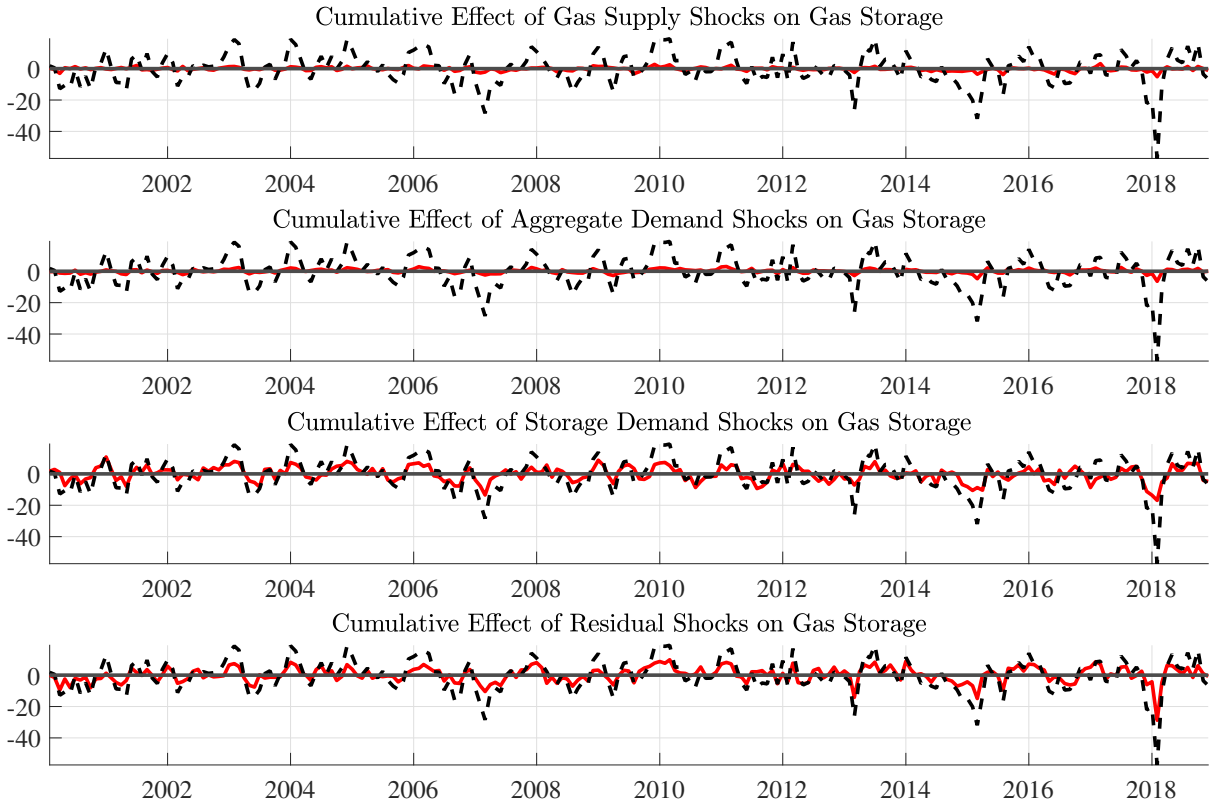

Figure A.5: Historical decomposition of changes in natural gas inventories for 2000:2–2018:12

Note: Dashed lines show deviations of actual data from the deterministic trend. Solid lines show the contribution of each shock to the deviation from the deterministic trend.

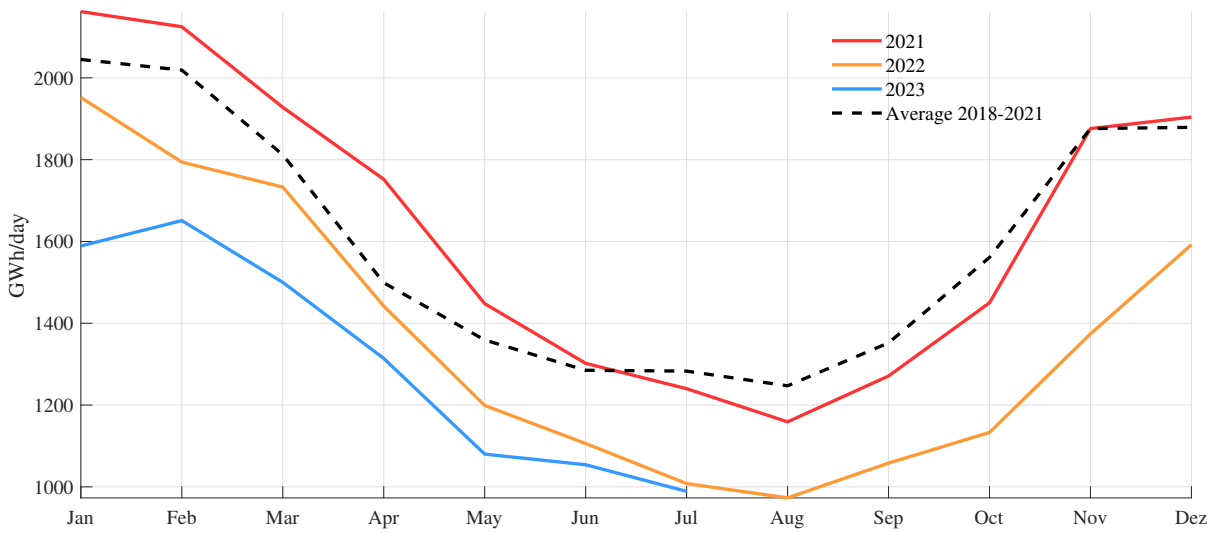

Figure A.6: Natural gas use by German industry in 2022 and 2023 relative to 2018–2021 average

Source: Bundesnetzagentur

A.2 Details on the Time-Varying Parameter VAR

Subsequently, we provide details on the quasi-Bayesian estimation of The TVP-SV-VAR following Petrova (2019). The model reads as follows:

$$y_t = c_t + \sum_{i=1}^{p} B_{i,t} y_{t-i} + u_t, \qquad u_t = R_t^{-1/2}\varepsilon_t, \qquad \varepsilon_t \sim N(0, I_N). \tag{A.1}$$

Defining $x_t = (1, y'_{t-1}, \ldots, y'_{t-p})$ and $B_t = (c_t, B_{1,t}, \ldots, B_{p,t})$ allows to write the model as:

$$y_t = (I_N \otimes x_t)\beta_t + R_t^{-1/2}\varepsilon_t, \tag{A.2}$$

$$\theta_t = \begin{bmatrix} \beta_t & vech(R_t^{-1/2}) \end{bmatrix}'. \tag{A.3}$$

If θ satisfies either of the following conditions, the sequence of time-varying parameters moves slowly over time, which is a sufficient property for consistent estimation:

(i) θ_t is a deterministic process $\theta_t = \theta(t|T)$, where $\theta(\cdot)$ is a piecewise differentiable function.

(ii) θ_t is a stochastic process satisfying: $\sup_{j:|j-t|<h}||\theta_t - \theta_j||^2 = O_p(h|t)$ for $1 \leq h \leq t$ for $t \to \infty$.

The local likelihood function of model (A.1) for each period j is given by:

$$\varphi_{T,j}(\theta_j) = \sum_{t=1}^{T} \vartheta_{j,t} l_t(y_t|y^{t-1}, \theta_j), \qquad \text{for } j, t \in \{1, \ldots, T\}, \tag{A.4}$$

where $l_t(y_t|y^{t-1}, \theta_j)$ is the conditional log-density for observation t and $\vartheta_{j,t}$ reweighs the likelihood of the observations $(y_1, \ldots, y_T)$. For $j, t \in \{1, \ldots, T\}$, these weights are computed using a kernel function:

$$\vartheta_{j,t} = \varkappa_{j,t}\omega_{j,t}, \quad \omega_{j,t} = \tilde{\omega}_{j,t} \sum_{t=T}^{T} \tilde{\omega}_{j,t}, \quad \tilde{\omega}_{j,t} = K\left(\frac{j-t}{H}\right), \quad \varkappa_{j,t} = \left(\sum_{t=1}^{T} \omega_{j,t}^2\right)^{-1}, \tag{A.5}$$

$K(\cdot)$ is a non-negative, continuous, and bounded kernel function with bandwidth parameter H, satisfying $H \to \infty$ and $H = o(T/\log T)$.[21] The kernel function reweighs the model's log-likelihood function at each period j. The decay of the weights is determined by H. The higher the value for H, the slower the weights of the weights decay. Combining the local likelihood function with a Normal-Wishart prior for β_j and R_j:

$$p(\beta_j|R_j) \sim N(\underline{\beta}_j, (R_j \otimes \underline{\kappa}_j)^{-1}), \qquad p(R_j) \sim W(\underline{\alpha}_j, \underline{\gamma}_j^{-1}), \qquad \text{for } j \in \{1, \ldots, T\}, \tag{A.6}$$

[21] In the case of a Normal kernel, the weights are given by: $\omega_{j,t} = \frac{1}{\sqrt{2\phi}} \exp((\frac{-1}{2})(j-t)/H^2)$ for $j, t \in \{1, \ldots, T\}$.

gives rise to a Normal-Wishart quasi-posterior for β_j and R_j:

$$p(\beta_j|R_j, X, Y) \sim N(\overline{\beta_j}, (R_j \otimes \overline{\kappa})^{-1}), \quad p(R_j|Y, X) \sim W(\overline{\alpha_j}, \overline{\gamma_j}^{-1}), \quad \text{for } j \in \{1, \ldots, T\}, \tag{A.7}$$

where, for $j \in \{1, \ldots, T\}$, the posterior parameters are defined as follows:

$$\overline{\beta}_j = (I_N \otimes \overline{\kappa}_j^{-1})[(I_N \otimes X'D_jX)\hat{\beta}_j + (I_N \otimes \underline{\kappa}_j)\underline{\beta}_j], \tag{A.8}$$

$$\overline{\kappa}_j = \underline{\kappa}_j + X'D_jX, \tag{A.9}$$

$$\overline{\alpha}_j = \underline{\alpha}_j + \sum_{t=1}^{T} \vartheta_{j,t}, \tag{A.10}$$

$$\overline{\gamma}_j = \underline{\gamma}_j + Y'D_jY + c_j\underline{\kappa}_jc_j' - \overline{B}_j\overline{\kappa}_j\overline{B}_j', \tag{A.11}$$

$$D_j = diag(\vartheta_{j,1}, \ldots, \vartheta_{j,T}), \tag{A.12}$$

and $\hat{\beta}_j = (I_N \otimes X'D_jX)^{-1}(I_N \otimes X'D_j)y$ is the local likelihood estimator for β_j derived by Giraitis et al. (2014).