

On the Exploitation of the Redundant Energy in UW-OFDM: LMMSE versus Sphere Detection

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Abstract—Unique word orthogonal frequency division multiplexing (UW-OFDM) inherently introduces a complex number Reed Solomon (RS) code. Originally, the code generator matrix of systematic coded UW-OFDM had been designed rather intuitively by minimizing the mean redundant energy. In this work we justify this approach by applying a cost function that incorporates the overall transceiver chain including a linear minimum mean square error (LMMSE) data estimator. In addition to the LMMSE estimator we investigate a nonlinear sphere detection (SD) receiver for both systematic and nonsystematic coded UW-OFDM. We study and interpret the estimators' performance and their diverse ability to exploit the redundant energy.

Index Terms—OFDM, Unique word OFDM (UW-OFDM), Cyclic prefix (CP), Minimum mean square error (MMSE), Sphere detection (SD).

I. INTRODUCTION

In [1]-[3] we introduced an orthogonal frequency division multiplexing (OFDM) signaling scheme, where the usual cyclic prefixes (CPs) are replaced by deterministic sequences, that we call unique words (UWs). Different as in KSP (known symbol padding)-OFDM [4] the UWs are part of the discrete Fourier transform (DFT)-interval, which requires a certain level of redundancy in frequency domain. In [1] we proposed to generate UW-OFDM symbols by appropriately loading a set of dedicated redundant subcarriers. This process introduces a systematic Reed Solomon (RS) code over the field of complex numbers (instead of a finite field as usual). We optimized the positions of the redundant subcarriers by minimizing their mean energy contribution which leads to an improved bit error ratio (BER) performance. However, this original UW-OFDM concept still suffers from a disproportionately high energy contribution of the redundant subcarriers. In [5] we solved this problem by introducing a nonsystematic complex number RS code construction. The idea of dedicated redundant subcarriers is abandoned, and the redundancy is distributed across all subcarriers. In [5], the code generator matrix has been chosen to

This work was supported by the Austrian Science Fund (FWF): I683-N13 and by the German Research Foundation (DFG): HU 634/9-1.

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Digital Object Identifier 10.1109/LSP.2012.2194737

be optimally matched to the linear minimum mean square error (LMMSE) data estimation procedure. Nonsystematic coded UW-OFDM in combination with LMMSE data estimation has been shown in [5] to significantly outperform classical OFDM and the original systematic coded UW-OFDM.

In the present paper we show, that minimizing the mean redundant energy in systematic coded UW-OFDM is in fact also optimum in the sense that the sum of the error variances after an LMMSE data estimation is minimized. Furthermore, we compare the LMMSE estimator with a sphere detector (SD) for both systematic and nonsystematic coded UW-OFDM. It turns out, that under AWGN conditions the SD optimally exploits the excess of redundant energy in systematic coded UW-OFDM, and it asymptotically reaches the performance of nonsystematic coded UW-OFDM, for which the LMMSE estimator and the SD perform equivalently. In frequency selective environments nonsystematic coded UW-OFDM in combination with an SD inherently exploits the diversity offered by the channel most effectively.

II. REVIEW OF UW-OFDM

A. Transmit Symbol Generation

In the following we use a tilde to express frequency domain vectors and matrices ($\tilde{\mathbf{a}}, \tilde{\mathbf{A}}, \dots$), respectively. Let $\mathbf{x}_u \in \mathbb{C}^{N_u \times 1}$ be a predefined sequence which we call unique word. This unique word shall form the tail of each OFDM time domain symbol vector. Hence, an UW-OFDM time domain symbol vector of length N consists of two parts and is of the form $[\mathbf{x}_d^T \ \mathbf{x}_u^T]^T$, at which only $\mathbf{x}_d \in \mathbb{C}^{(N-N_u) \times 1}$ is random and affected by the data. Following [2], we generate the time domain symbol $\mathbf{x} = [\mathbf{x}_d^T \ \mathbf{0}^T]^T$ with a zero UW in a first step, and we determine the final transmit symbol $\mathbf{x}' = \mathbf{x} + [\mathbf{0}^T \ \mathbf{x}_u^T]^T$ by adding the desired UW in time domain in a second step. As in conventional OFDM, the QAM data symbols (denoted by the vector $\tilde{\mathbf{d}} \in \mathbb{C}^{N_d \times 1}$) and the zero subcarriers (usually at the band edges and at DC) are specified as part of the frequency domain vector $\tilde{\mathbf{x}}$, but here in addition the zero-word is specified in time domain as part of the vector $\mathbf{x} = \mathbf{F}_N^{-1} \tilde{\mathbf{x}}$. \mathbf{F}_N denotes the length- N -DFT matrix with elements $[\mathbf{F}_N]_{kl} = e^{-j\frac{2\pi}{N}kl}$ for $k, l = 0, 1, \dots, N-1$. The generation of the zeros in the time domain requires a certain level of redundancy in the frequency domain. For this purpose we define codewords $\tilde{\mathbf{c}} \in \mathbb{C}^{(N_d+N_r) \times 1}$ with $N_r = N_u$ by

$$\tilde{\mathbf{c}} = \mathbf{G} \tilde{\mathbf{d}}, \quad (1)$$

where $\mathbf{G} \in \mathbb{C}^{(N_d+N_r) \times N_d}$ depicts a complex valued code generator matrix. Furthermore, we model the insertion of the zero subcarriers by $\tilde{\mathbf{x}} = \mathbf{B}\tilde{\mathbf{c}}$, where $\mathbf{B} \in \{0, 1\}^{N \times (N_d+N_r)}$ consists of zero-rows at the positions of the zero subcarriers, and of appropriate unit row vectors at the positions of occupied subcarriers. With these definitions the system of equations $\mathbf{F}_N^{-1}\tilde{\mathbf{x}} = \mathbf{x}$ takes on the form $\mathbf{F}_N^{-1}\mathbf{B}\mathbf{G}\tilde{\mathbf{d}} = [\mathbf{x}_d^T \ \mathbf{0}^T]^T$. Consequently, in order that the zero UW is generated for every possible data vector $\tilde{\mathbf{d}}$, \mathbf{G} has to fulfill the constraint

$$\mathbf{F}_N^{-1}\mathbf{B}\mathbf{G} = \begin{bmatrix} * \\ \mathbf{0} \end{bmatrix}. \quad (2)$$

With the frequency domain version of the UW $\tilde{\mathbf{x}}_u = \mathbf{F}_N [\mathbf{0}^T \ \mathbf{x}_u^T]^T$ the transmit symbol can finally be written as

$$\mathbf{x}' = \mathbf{F}_N^{-1}(\mathbf{B}\mathbf{G}\tilde{\mathbf{d}} + \tilde{\mathbf{x}}_u). \quad (3)$$

In our original UW-OFDM concept in [1]-[2] we chose

$$\mathbf{G} = \mathbf{P} \begin{bmatrix} \mathbf{I} \\ \mathbf{T} \end{bmatrix} \quad (4)$$

with a carefully selected permutation matrix $\mathbf{P} \in \{0, 1\}^{(N_d+N_r) \times (N_d+N_r)}$ and with $\mathbf{T} \in \mathbb{C}^{N_r \times N_d}$. In this approach the frequency domain symbol follows to $\tilde{\mathbf{x}} = \mathbf{B}\mathbf{G}\tilde{\mathbf{d}} = \mathbf{B}\mathbf{P} [\tilde{\mathbf{d}}^T \ \tilde{\mathbf{r}}^T]^T$, where the vector of *dedicated* redundant subcarriers is given by $\tilde{\mathbf{r}} = \mathbf{T}\tilde{\mathbf{d}}$. By using (4) the constraint in (2) can be re-written as

$$\mathbf{F}_N^{-1}\mathbf{B}\mathbf{P} \begin{bmatrix} \mathbf{I} \\ \mathbf{T} \end{bmatrix} = \begin{bmatrix} * \\ \mathbf{0} \end{bmatrix}. \quad (5)$$

With $\mathbf{M} = \mathbf{F}_N^{-1}\mathbf{B}\mathbf{P} = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{bmatrix}$, where \mathbf{M}_{ij} are appropriately sized sub-matrices, (5) is fulfilled by choosing $\mathbf{T} = -\mathbf{M}_{22}^{-1}\mathbf{M}_{21}$. However, the choice of the permutation matrix \mathbf{P} which defines the positions of the dedicated data and redundant subcarriers turns out to be a highly critical design aspect. We will discuss this problem in detail in Sec. II-C. We further note, that \mathbf{G} as in (4) can be interpreted as the code generator matrix of a systematic Reed Solomon code over the field of complex numbers (instead of a finite field as usual), cf. [5].

B. Data Estimation

After the transmission over a dispersive channel a received frequency domain UW-OFDM symbol (after elimination of the zero subcarriers) can be modeled as

$$\tilde{\mathbf{y}}_d = \tilde{\mathbf{H}}\mathbf{G}\tilde{\mathbf{d}} + \tilde{\mathbf{H}}\mathbf{B}^T\tilde{\mathbf{x}}_u + \mathbf{B}^T\mathbf{F}_N\mathbf{n}, \quad (6)$$

cf. (3). A detailed derivation of (6) can e.g. be found in [5]. $\tilde{\mathbf{H}} \in \mathbb{C}^{(N_d+N_r) \times (N_d+N_r)}$ denotes the diagonal channel matrix which contains the sampled channel frequency response on its main diagonal, and $\mathbf{n} \in \mathbb{C}^{N \times 1}$ represents a zero-mean Gaussian (time domain) noise vector with covariance matrix $\sigma_n^2\mathbf{I}$. Note that $\tilde{\mathbf{H}}\mathbf{B}^T\tilde{\mathbf{x}}_u$ represents a known portion contained in the received vector $\tilde{\mathbf{y}}_d$ originating from the UW. As a first step of the receiver processing we therefore subtract the UW influence (assuming that the channel matrix $\tilde{\mathbf{H}}$ or at least an estimate of it is available) to obtain the corrected symbol $\tilde{\mathbf{y}} = \tilde{\mathbf{y}}_d - \tilde{\mathbf{H}}\mathbf{B}^T\tilde{\mathbf{x}}_u$ in the form of the linear model

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\mathbf{G}\tilde{\mathbf{d}} + \tilde{\mathbf{v}}, \quad (7)$$

with the noise vector $\tilde{\mathbf{v}} = \mathbf{B}^T\mathbf{F}_N\mathbf{n}$. $\tilde{\mathbf{y}}$ serves as the input of the data estimation procedure. The most common data estimator is the linear minimum mean square error estimator. The LMMSE data estimate can be found to be

$$\hat{\tilde{\mathbf{d}}}_{\text{LMMSE}} = (\mathbf{G}^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{G} + \frac{N\sigma_n^2}{\sigma_d^2}\mathbf{I})^{-1}\mathbf{G}^H\tilde{\mathbf{H}}^H\tilde{\mathbf{y}}, \quad (8)$$

where a zero-mean QAM data vector with the covariance matrix $\sigma_d^2\mathbf{I}$ is assumed, cf. [3]. The covariance matrix of the error $\tilde{\mathbf{e}} = \tilde{\mathbf{d}} - \hat{\tilde{\mathbf{d}}}$ is

$$\mathbf{C}_{\tilde{\mathbf{e}}\tilde{\mathbf{e}}} = N\sigma_n^2(\mathbf{G}^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{G} + \frac{N\sigma_n^2}{\sigma_d^2}\mathbf{I})^{-1}. \quad (9)$$

The overall transceiver performance can generally further be improved by applying nonlinear data estimation principles. For equiprobable data sequences an optimum receiver is the maximum likelihood sequence estimator (MLSE) which selects the data vector that minimizes the Euclidean distance between the actually received vector $\tilde{\mathbf{y}}$ and each possible noise free receive symbol vector (for a given $\tilde{\mathbf{H}}$):

$$\hat{\tilde{\mathbf{d}}}_{\text{MLSE}} = \arg \min_{\tilde{\mathbf{d}} \in \mathcal{A}^{N_d}} \left\| \tilde{\mathbf{H}}\mathbf{G}\tilde{\mathbf{d}} - \tilde{\mathbf{y}} \right\|_2^2 \quad (10)$$

Here, \mathcal{A} denotes the chosen QAM alphabet. The MLSE solution can efficiently (on average) be implemented with the sphere detection approach which is a well known method usually applied in multiple antenna systems. To enable SD, a QR-decomposition $\tilde{\mathbf{H}}\mathbf{G} = \mathbf{Q} \begin{bmatrix} \mathbf{R}_0 \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_1 & \mathbf{Q}_2 \end{bmatrix} \begin{bmatrix} \mathbf{R}_0 \\ \mathbf{0} \end{bmatrix}$, with unitary $\mathbf{Q} \in \mathbb{C}^{(N_d+N_r) \times (N_d+N_r)}$ and upper triangular $\mathbf{R} \in \mathbb{C}^{N_d \times N_d}$ is required. It can immediately be shown that the optimization problem in (10) can be re-written as

$$\hat{\tilde{\mathbf{d}}}_{\text{MLSE}} = \arg \min_{\tilde{\mathbf{d}} \in \mathcal{A}^{N_d}} \left\| \mathbf{R}\tilde{\mathbf{d}} - \tilde{\mathbf{y}}' \right\|_2^2, \quad (11)$$

where $\tilde{\mathbf{y}}' = \mathbf{Q}_1^H\tilde{\mathbf{y}}$. The representation in (11) allows the application of the tree-search based SD approach. In this work we used a slightly adapted version of the Schnorr-Euchner strategy described in [6].

C. Finding the Optimum Permutation Matrix

In [1] we suggested to choose the permutation matrix \mathbf{P} such that the mean redundant energy becomes minimum. To motivate this approach we first discuss the mean OFDM symbol energy $E_{\mathbf{x}'} = E[\mathbf{x}'^H\mathbf{x}']$ which can easily be calculated to

$$E_{\mathbf{x}'} = \frac{\sigma_d^2}{N} \text{tr}\{\mathbf{G}^H\mathbf{G}\} + \mathbf{x}_u^H\mathbf{x}_u \quad (12)$$

$$= \frac{1}{N} \left(\underbrace{N_d\sigma_d^2}_{E_{\tilde{\mathbf{d}}}} + \underbrace{\sigma_d^2\text{tr}\{\mathbf{T}\mathbf{T}^H\}}_{E_{\tilde{\mathbf{r}}}} \right) + \underbrace{\mathbf{x}_u^H\mathbf{x}_u}_{E_{\mathbf{x}_u}}. \quad (13)$$

In the latter step we used $\mathbf{G}^H\mathbf{G} = \mathbf{I} + \mathbf{T}^H\mathbf{T}$, cf. (4). $\frac{E_{\tilde{\mathbf{d}}}}{N}$ and $\frac{E_{\tilde{\mathbf{r}}}}{N}$ describe the contributions of the data and the redundant subcarrier symbols to the mean transmit symbol energy before the addition of the UW, respectively, and $E_{\mathbf{x}_u}$ describes the contribution of the UW. Note that \mathbf{T} and thus also $E_{\tilde{\mathbf{r}}}$ depend on \mathbf{P} . $E_{\tilde{\mathbf{r}}}$ can take on extremely high values for inappropriate

choices of \mathbf{P} , a disadvantageous option is e.g. $\mathbf{P} = \mathbf{I}$. In [1] we therefore decided to choose \mathbf{P} by minimizing the cost function

$$J_E = \frac{E_{\bar{r}}}{N} = \frac{\sigma_d^2}{N} \text{tr} \{ \mathbf{T} \mathbf{T}^H \}. \quad (14)$$

We solved this discrete optimization problem by means of a heuristic approach. The solver computes the optimum \mathbf{P} within a few seconds (of course depending on N) using e.g. Matlab on a standard PC. In Sec. IV we give an example of the redundant subcarrier distribution that minimizes J_E for a specific parameter setup.

The BER simulation results in [1] confirmed that this approach leads to an excellent system performance. Nevertheless, the cost function J_E only takes the transmit symbols' (mean) energy into account and the question arises whether this choice is effectively optimum in terms of the overall transceiver performance. We now regard a different cost function which focuses on the system performance including the data estimation at the receiver. We assume that the LMMSE estimator is used since its error covariance matrix is available in closed form. A possible measure of the overall system performance is the sum of the error variances after the data estimation. From (9) it becomes clear that this measure would depend on the particular channel instance $\tilde{\mathbf{H}}$. We are aiming to design the code generator matrix \mathbf{G} (which is unambiguously defined by the choice of \mathbf{P}) only once during system design, and we therefore look for a cost function for $\tilde{\mathbf{H}} = \mathbf{I}$, that is the AWGN channel case. With (9) the sum of the error variances then becomes

$$\text{tr} \{ \mathbf{C}_{\tilde{e}\tilde{e}} \} = N \sigma_n^2 \text{tr} \left\{ \left(\mathbf{G}^H \mathbf{G} + \frac{N \sigma_n^2}{\sigma_d^2} \mathbf{I} \right)^{-1} \right\}. \quad (15)$$

Let $E_s = \frac{E_{s'}}{N_d}$ denote the mean energy per data symbol. For a fair performance comparison of different code generator matrices \mathbf{G} (or equivalently of different matrices \mathbf{P}) we fix the ratio $c = \frac{E_s}{\sigma_n^2}$ during the optimization. By applying (12) with the assumption of a zero UW our cost function finally reads

$$J_{\text{LMMSE}} = \sigma_d^2 \text{tr} \left\{ \left(\frac{c N_d}{\text{tr} \{ \mathbf{G}^H \mathbf{G} \}} \mathbf{G}^H \mathbf{G} + \mathbf{I} \right)^{-1} \right\} \quad (16)$$

$$= \sigma_d^2 \text{tr} \left\{ \left(\frac{c N_d}{N_d + \text{tr} \{ \mathbf{T} \mathbf{T}^H \}} (\mathbf{T}^H \mathbf{T} + \mathbf{I}) + \mathbf{I} \right)^{-1} \right\}. \quad (17)$$

For the minimization of J_{LMMSE} as in (17) we can use the same heuristic solver as for the minimization of J_E , we only have to exchange the cost function. For the example in Sec. IV optimum permutation matrices have been determined by minimizing J_E and J_{LMMSE} , respectively. It is of great interest that solving the two different optimization problems leads to the same \mathbf{P} . Apparently, the more intuitive approach of minimizing the mean redundant energy which has been used in [1]-[2] was in fact a highly reasonable choice.

III. NONSYSTEMATIC CODED UW-OFDM

Recently, in [5] we introduced the so-called nonsystematic coded UW-OFDM concept, where we proposed a code generator matrix $\check{\mathbf{G}}$ that allows to distribute the redundancy over

all subcarriers instead of only dedicated ones. For that purpose we model the code generator matrix as

$$\check{\mathbf{G}} = \mathbf{A} \begin{bmatrix} \mathbf{I} \\ \check{\mathbf{T}} \end{bmatrix}, \quad (18)$$

where the non-singular real matrix $\mathbf{A} \in \mathbb{R}^{(N_d+N_r) \times (N_d+N_r)}$ replaces \mathbf{P} , cf. (4). Thus, the constraint in (2) becomes

$$\mathbf{F}_N^{-1} \mathbf{B} \mathbf{A} \begin{bmatrix} \mathbf{I} \\ \check{\mathbf{T}} \end{bmatrix} = \begin{bmatrix} * \\ \mathbf{0} \end{bmatrix}. \quad (19)$$

With $\check{\mathbf{M}} = \mathbf{F}_N^{-1} \mathbf{B} \mathbf{A} = \begin{bmatrix} \check{\mathbf{M}}_{11} & \check{\mathbf{M}}_{12} \\ \check{\mathbf{M}}_{21} & \check{\mathbf{M}}_{22} \end{bmatrix}$ the constraint in (19) is fulfilled by choosing $\check{\mathbf{T}} = -\check{\mathbf{M}}_{22}^{-1} \check{\mathbf{M}}_{21}$. In [5] we aimed at finding a generator matrix $\check{\mathbf{G}}$ that minimizes the sum of the error variances after LMMSE data estimation for $\tilde{\mathbf{H}} = \mathbf{I}$ at a fixed $c = \frac{E_s}{\sigma_n^2}$. This clearly leads to the same cost function as in (16), we only have to substitute \mathbf{G} by $\check{\mathbf{G}}$. Note, that J_{LMMSE} has now to be treated as a function of the real valued matrix \mathbf{A} instead of the permutation matrix \mathbf{P} . The solution to this optimization problem is ambiguous. In [5] we have shown that every solution fulfills $\check{\mathbf{G}}^H \check{\mathbf{G}} = s^2 \mathbf{I}$, where s corresponds to the all identical singular values of $\check{\mathbf{G}}$. Furthermore, and different to the systematic coded case the error covariance matrix after LMMSE data estimation in the AWGN channel becomes diagonal:

$$\mathbf{C}_{\tilde{e}\tilde{e}} = \frac{\sigma_d^2}{c+1} \mathbf{I} \quad (20)$$

Particular solutions of the optimization problem can e.g. be found by applying the steepest descent algorithm. In [5] we chose the initialization $\mathbf{A}^{(0)} = \mathbf{P}$ which implies $\check{\mathbf{T}}^{(0)} = \mathbf{T}$ and $\check{\mathbf{G}}^{(0)} = \mathbf{P} \begin{bmatrix} \mathbf{I} & \mathbf{T}^T \end{bmatrix}^T = \mathbf{G}$. The iterative optimization process consequently starts with the code generator matrix \mathbf{G} of the systematic coded UW-OFDM concept, which can be assumed to be a good initial guess. In correspondence to [5] we denote the resulting optimum code generator matrix with $\check{\mathbf{G}}'$.

IV. SIMULATION RESULTS

The parameters of our simulated system are adapted to current wireless local area network (WLAN) standards and are as follows: $N = 64$, $N_d = 36$, $N_r = N_u = 16$, sampling frequency $f_s = 20\text{MHz}$, DFT period $T_{\text{DFT}} = 3.2\mu\text{s}$, guard duration $T_{\text{GI}} = 800\text{ns}$. The index set of the zero subcarriers is $\{0, 27, 28, \dots, 37\}$, and the optimum index set for the redundant subcarriers minimizing J_E is $\{2, 6, 10, 14, 17, 21, 24, 26, 38, 40, 43, 47, 50, 54, 58, 62\}$. This choice can easily also be described by appropriate matrices \mathbf{P} and \mathbf{B} , respectively. It is a highly interesting observation that this choice of positions of the redundant subcarriers also minimizes J_{LMMSE} in (17). The upper plot in Fig. 1 shows the mean power distribution over the individual subcarriers for this parameter setup in case the zero UW is used (which will be the case in all subsequent simulation results). The optimized mean power values of the redundant subcarrier symbols are the elements of the vector $\sigma_d^2 \text{diag}(\mathbf{T} \mathbf{T}^H)$ evaluated for the optimum \mathbf{P} , cf. (13). It can be observed that the mean power of the redundant subcarrier symbols is still considerably higher than that of the data symbols ($\sigma_d^2 = 1$). The lower plot in Fig. 1 shows the mean

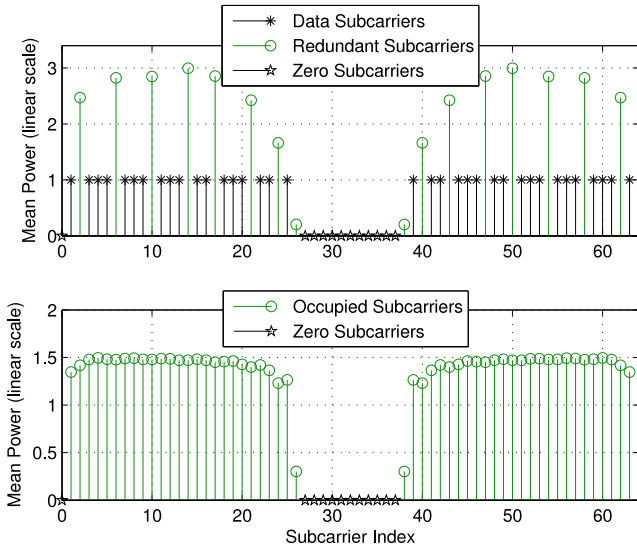


Fig. 1. Mean power of individual subcarrier symbols for \mathbf{G} (systematic coded UW-OFDM; above), and \mathbf{G}' (nonsystematic coded UW-OFDM; below).

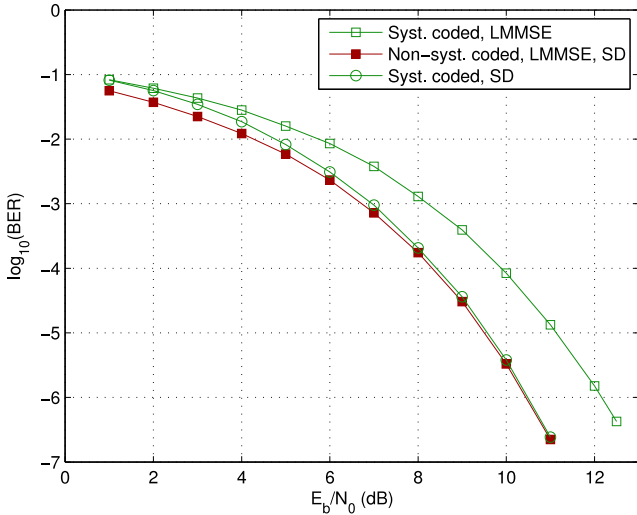


Fig. 2. BER simulation results in the AWGN channel.

power distribution for the nonsystematic coded UW-OFDM system, where \mathbf{G} has been replaced by \mathbf{G}' . We can clearly identify that \mathbf{G}' implicates a significant mean power reduction for subcarriers that corresponded to redundant symbols in the original UW-OFDM approach. Furthermore, it can be seen that the redundant energy is now smeared over all subcarriers.

Fig. 2 shows BER simulation results for the AWGN channel (modulation: QPSK). We can observe, that in case the LMMSE data estimator is used \mathbf{G}' (nonsystematic RS code) clearly outperforms \mathbf{G} (systematic RS code), the gain at a BER of 10^{-6} is 1.7dB. For \mathbf{G}' the SD and the LMMSE receiver show the same performance. Consequently, in the AWGN channel case the simple LMMSE estimator already represents the optimum data estimator for nonsystematic coded UW-OFDM. On the other hand, for the systematic coded system the SD data estimator significantly outperforms the LMMSE estimator. The SD is able to exploit the excess of redundant energy provided by \mathbf{G} compared to \mathbf{G}' . It is exciting to observe, that the performance of the systematic coded system in combination

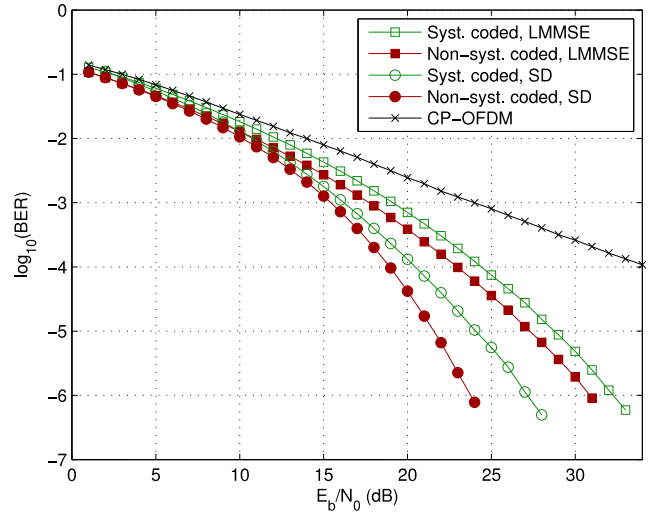


Fig. 3. BER simulation results in a frequency selective indoor environment.

with the SD receiver asymptotically reaches the performance of the nonsystematic coded system.

Fig. 3 shows results for the frequency selective case. The indoor channel impulse responses are modeled as tapped delay lines, each tap with uniformly distributed phase and Rayleigh distributed magnitude, and with power decaying exponentially, cf. [7]. For each BER curve we averaged over 10000 random channel realizations, featuring (on average) an rms delay spread of 100ns, and all being normalized such that the receive power is independent of the actual channel. Perfect channel knowledge is assumed at the receiver. For the LMMSE estimator the gain of \mathbf{G}' over \mathbf{G} is comparable to the AWGN channel result. However, different to the AWGN case, the SD significantly improves the performance over the LMMSE estimator for both, \mathbf{G}' and \mathbf{G} . For the SD receiver \mathbf{G}' outperforms \mathbf{G} by 3.5dB at a BER of 10^{-6} . Nonsystematic coded UW-OFDM in combination with an SD consequently exploits the diversity offered by frequency selective channels most effectively.

Finally, it is worth to compare the UW-OFDM concepts with traditional CP-OFDM. Clearly, UW-OFDM shows an increased computational complexity, cf. [3], however, due to the inherent RS code all discussed UW-OFDM versions significantly outperform a comparable CP-OFDM system (Fig. 3 shows a simulation result of the CP-OFDM based IEEE 802.11a WLAN standard without outer channel coding), while featuring an (almost) identical bandwidth efficiency.

V. CONCLUSION

In this work we compared the LMMSE data estimator and the SD receiver for systematic and nonsystematic complex number RS coded UW-OFDM systems. The characteristics of the two estimators are discussed for the AWGN as well as for frequency selective channels. Furthermore, our original code generator matrix design approach for systematic coded UW-OFDM is justified by the introduction and minimization of a different cost function which focuses on the overall transceiver performance rather than on the redundant energy.

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