In the first part of this research proposal, we want to investigate finite-dimensional subspaces of L1, i.e., the so-called local structure of this space, and related topics, e.g. combinatorial and probabilistic inequalities. To be more precise, we want to study certain classes of finite-dimensional sequence spaces, i.e., generalized Orlicz spaces such as Musielak-Orlicz spaces, Orlicz-Lorentz spaces and Musielak-Orlicz-Lorentz spaces. The goal is to find easily verifiable conditions to decide wether a given Banach space is a subspace of L1 or not.

The methods in our approach involve combinatorial and probabilistic inequalities in connection with Orlicz norms. Hence, throughout the proposal, we will take a closer look at those inequalities. In this context, we also want to prove some inversion formulas, telling us, given a certain Orlicz function, how to choose the distribution of the random variables so that the probabilistic expression is equivalent to the given Orlicz norm. Such inversion results immediately provide direct embeddings of certain Orlicz spaces into L1.

In recent years, more and more applications of those inequalities appeared, e.g., non-parametric statistics, random matrix theory, convex geometry. In fact, in the third part, we will also take a look at applications in convex geometry. We will study the expectation of support functions, the mean width of random polytopes and their perturbations and the so-called mean outer radii of random polytopes. The probabilistic expressions do appear naturally in this context.