

The goal of the project is to obtain asymptotics of polynomials which are orthogonal on the unit circle (abbreviated OPUC) on general sets and with respect to general measures. By general sets we mean so-called homogenous sets which include for instance thick Cantor type sets and general measures are supposed to have a "strong" absolutely continuous spectrum and, in particular, may have an infinite number of mass points. We expect that for such sets and measures the associated CMV matrices and hence the Verblunsky coefficients are asymptotically almost periodic. In other words we will work with operators close to a CMV matrix with constant coefficients, but as well with operators that are close to the isospectral set of almost periodic CMV matrices, possibly with a Cantor type spectrum. To simplify things we demonstrate the approach we have in mind for the case when the spectrum is a single arc: First we represent CMV matrices with constant coefficients as a multiplication operator in  $L^2$ -space with respect to a specific basis. This basis substitutes the standard basis in  $L^2$ , which is used for the free Jacobi matrix. Then we want to demonstrate that a similar orthonormal system in a certain "weighted" Hilbert space, which we call the Fadeev-Marchenko (FM) space, behaves asymptotically as the system in the standard (free) case discussed just before. The duality between the two types of Hardy subspaces in it, as expected, will play the key role in the proof of all asymptotics involved. Thus, our preparations show that a solution of the problem should be possible by developing a modern, extended scattering theory, interesting for itself. More precisely we believe that the traditional (Faddeev-Marchenko) condition is too restrictive to define the class of CMV matrices for which there exists a unique scattering representation. The main conjectures are: 1) Szegő-Blaschke class: the class of twosided CMV matrices acting in  $l^2$ , whose spectral density satisfies the Szegő condition and whose point spectrum the Blaschke condition, corresponds precisely to the class where the scattering problem can be posed and solved. That is, to a given CMV matrix of this class, one can associate the scattering data and related to them the FM space. The CMV matrix corresponds to the multiplication operator in this space, and the orthonormal basis in it (corresponding to the standard basis in  $l^2$ ) behaves asymptotically as the basis associated with the free system. 2)  $A_2$ -Carleson class: from the point of view of the scattering problem, the most natural class of CMV matrices is that one in which a) the scattering data determine the matrix uniquely and b) the associated Gelfand-Levitan Marchenko transformation operators are bounded. Necessary and sufficient conditions for this class can be given in terms of an  $A_2$  kind condition for the density of the absolutely continuous spectrum and a Carleson kind condition for the discrete spectrum.