

The proposal "Spectral Theory, Abelian Coverings and Iterations" deals with spectral theory of ergodic difference/differential operators of the second order. The importance of the topic was recently recognized at the highest level by the mathematical community: Avila devoted his whole Fields Medal talk to quasi-periodic Schrödinger operators. Roughly speaking the theory uses direct and inverse methods in its development. Both approaches have their positive and negative aspects, weak and strong sides. In a broad sense the goal of the project is to construct a quite comprehensive theory for classes of operators that are typical in the direct spectral theory based on the inverse spectral theory approach. As a sample of a comprehensive theory we can mention our joint with Volberg result on operators with absolutely continuous spectrum (Kotani-Last problem). In this case we are looking for a theory of such operators with singular spectrum, and more specifically, we would like to clarify up to which extend an essential dependence of frequencies (quasi- and limit-periodic classes of operators) is important to observe this spectral phenomena. Even in this clarification the goal is very ambitious. Indeed, we know that every reflectionless Jacobi matrix whose spectrum is a Cantor set of positive Lebesgue measure is almost periodic, but "what one can say if the spectrum is the classical Cantor set?" (Carleson). Here is a quotation from a recent paper by Krüger and Simon: "... in this case, we mainly have conjectures, discussion, and some numerical experiments..." At the moment we restrict ourselves by two basic ideas: (1) to use Abelian Coverings and related functional spaces of analytic functions in application to ergodic operators with singular continuous spectrum; (2) to use GMP matrices and iteration theory for rational functions to study properties of Jacobi matrices associated to the corresponding Julia sets. The first idea deals with a well-known result of Lyons and McKean, who demonstrated a dramatic extension of allowable classes of analytic functions on Riemann surfaces in passing to their Abelian Coverings. GMP matrices were recently found and used successfully to solve Killip-Simon problem. Note that goal (2) can be considered as a certain intermediate case between a comparably well-studied (in this setting) problem on Julia sets associated to iterations of polynomials (Bellissard, Bessis, Moussa) and the mentioned Carleson question on the standard Cantor set. With respect to an analytic property that might play a crucial role in the general context, Sodin conjectured that the resolvent domain of an almost periodic family should be uniformly perfect. Pommerenke described properties of such sets in terms of averaging operators acting on a universal covering. In this way one gets rich families of character automorphic functions. Note that corresponding classes of analytic in the unit disc functions were studied by Korenblum and his followers.