Analysis Seminar 2018

8th – 10th June, Traunkirchen, Austria

Institut für Analysis, Altenberger Straße 69, 4040 Linz
https://www.jku.at/analysis/

Main speakers:
James Bell Cooper, Johannes Kepler University, Linz
Eva Kopecká, University of Innsbruck
Joscha Prochno, University of Hull
Jan Vybíral, Czech Technical University, Prague

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Symposion Hotel Post, Ortsplatz 5, 4801 Traunkirchen
https://www.hotel-post-traunkirchen.at/

Programme Schedule

Saturday 9th June, Morning Session	
09:00–09:45	Eva Kopecká, University of Innsbruck Geometry and convergence of products of orthogonal projections (p. 4)
10:00–10:20	Wolfgang Stockinger Johannes Kepler University, Linz Gaps, additive energy and Poissonian pair correlation problems (p. 7)
10:30–11:00	*Coffee break*
11:00–11:20	Richard Lechner, Johannes Kepler University, Linz On the dimension dependence of factorization problems (p. 4)
11:30–11:50	Tamas Titkos, Alfréd Rényi Institute of Mathematics, Budapest A unified approach to decompose positive elements (p. 7)
12:00–12:20	Tommaso Russo, Università degli Studi di Milano Separated families of unit vectors (p. 6)
12:30–12:50	Markus Passenbrunner, Johannes Kepler University, Linz Spline characterizations of the Radon-Nikodým-property (p. 5)
13:00	*Lunch*
Saturday 9th June, Afternoon Session	
15:30	*Coffee break*
18:00–18:45	Joscha Prochno, University of Hull The asymptotic volume ratio of Schatten p-classes (p. 6)
19:00-19:20	Christian Bargetz, University of Innsbruck On generic properties of nonexpansive mappings (p. 3)
19:30-19:50	Emir Medjic, University of Innsbruck On the convergence rate of the Alternating Method in Bachach spaces (p. 5)
20:00	*Dinner*
Sunday 10th June, Morning Session	
09:00–09:45	Jan Vybíral, Czech Technical University, Prague Approximation of structured functions and simple neural networks (p. 8)
10:00–10:20	Petro Yudytskiy, Johannes Kepler University, Linz KdV hierarchy via Abelian coverings and operator identities (p. 8)
10:30–11:00	*Coffee break*
11:00–11:20	Mario Ullrich, Johannes Kepler University, Linz The inverse of the dispersion depends logarithmically on the dimension (p. 7)
11:30–12:15	James Bell Cooper, Johannes Kepler University, Linz Analysis in the space of observables (p. 3)
12:30	*Lunch*

Abstracts

On generic properties of nonexpansive mappings

Christian Bargetz, University of Innsbruck

In the context of fixed point theory of nonexpansive mappings, F. S. de Blasi and J. Myjak came upon the question of whether the generic nonexpansive mapping on a convex, bounded and closed subset of a Banach space is a strict contraction, i.e., whether it is Lipschitz continuous with a Lipschitz constant strictly smaller than one.
We introduce a class of geodesic metric spaces and show that the generic nonexpansive mapping on star-shaped and closed subsets of these spaces even has local Lipschitz constant one at typical points of its domain. These results include the cases of nonexpansive self-mappings and the case of nonexpansive set-valued mappings.
We will also discuss some work in progress concerning the generic successive approximation of fixed points of certain set-valued mappings defined on a bounded, closed, and convex subset of a Banach space.
This is joint work with Michael Dymond and Simeon Reich.

Analysis in the space of observables

James Bell Cooper, Johannes Kepler University, Linz

Unbounded self-adjoint operators play the role of the real numbers in quantum mechanics, normal operators that of complex ones. Key tools of theoretical physicists in this context are distributions with values in the space of observables (quantum field theory) and analytic functions therein (perturbation methods). Despite this fact the literature provides no rigorous definition of these concepts or (at best) woefully inadequate ones. In our talk we present a theoretical basis for a rigorous approach to spaces of observable-valued tempered distributions and holomorphic functions. We show how the regular toolbox of theoretical physics (Fourier transform, Hermite expansions, McLaurin series) carries over to this situation and use this to display some non-trivial examples.

Geometry and convergence of products of orthogonal projections

Eva Kopecká, University of Innsbruck

Suppose $L_1, \dots, L_K$ are closed subspaces of a Hilbert space H and P_{L_i} are the orthogonal projections onto them. It is known that the cyclic product $(P_K \dots P_1)^n x$ converges for each starting point $x \in H$. Moreover, there is a dichotomy: the product converges exponentially fast iff $L_1^\perp + \dots + L_K^\perp$ is closed; otherwise it converges as slow as one likes for an appropriately chosen initial vector $x \in H$. Deutsch and Hundal asked if the closedness of $L_1^\perp + \dots + L_K^\perp$ implies the convergence of $z_n = P_{L_{k_n}} z_{n-1}$ for any starting point $z \in H$ and any sequence $k_1, k_2, \dots \in \{1, \dots, K\}$. We give an example that for $K \geq 5$ this is not the case. For $K = 3$ and $K = 4$ we show that $L_1^\perp + \dots + L_K^\perp$ is closed if and only if $z_n = P_{\tilde{L}_{k_n}} z_{n-1}$ converges for each $z \in H$, all closed subspaces $\tilde{L}_i \subset L_i$ and all sequences $k_1, k_2, \dots \in \{1, \dots, K\}$. We also explain what happens when $k \geq 5$.

On the dimension dependence of factorization problems

Richard Lechner, Johannes Kepler University, Linz

We discuss recent results on the dimension dependence of factorization problems in one- and two-parameter dyadic Hardy spaces. We improve previous super-exponential estimates to polynomial estimates.

References

R. Lechner. Dimension dependence of factorization problems: Hardy spaces and SL_n^∞. ArXiv e-prints https://arxiv.org/abs/1802.02857, Feb. 2018.

R. Lechner. Dimension dependence of factorization problems: bi-parameter Hardy spaces. ArXiv e-prints https://arxiv.org/abs/1802.05994, Feb. 2018.

On the convergence rate of the Alternating Method in Banach spaces

Emir Medjic, University of Innsbruck

Let X be a Banach space and M, N be closed subspaces, such that their sum $M+N$ is closed. A method to find the projection of a point $x \in X$ onto $M+N$ is the alternating approximation method (AAM), studied for example by F. Deutsch and A. Pinkus, see [1] and [2]. The (AAM) for $x \in X$ and an iteration step $n \in N$, is given by

$$((I - P_M)(I - P_N))^n x.$$

For uniformly convex and uniformly smooth Banach spaces this sequence converges and

$$\lim_{n\to\infty} ((I - P_M)(I - P_N))^n x = P_{\overline{M+N}} x,$$

for a proof see for example [1]. This result was extended in [2] to an arbitrary nite number of subspaces instead of only two subspaces M, N. We present a convergence rate for the above method in Banach spacces which are convex and smooth of power type. Furthermore, due to the intimate connection between metric and Bergman projections, we present results on the convergence rate of iterated Bregman projections in such spaces.
This is joint work with Christian Bargetz and still in progress.

References

[1] Frank Deutsch. *The Alternating Method von Neumann*, pages 83–96. Birkhäuser Basel, Basel, 1979.
[2] Allan Pinkus. The alternating algorithm in a uniformly convex and uniformly smooth Banach space, 01 2015

Spline characterizations of the Radon-Nikodým-property

Markus Passenbrunner, Johannes Kepler University, Linz

A Banach space X is said to have the Radon-Nikodým-property (RNP) if, for measures with values in X, the Radon-Nikodým theorem is true, i.e. if for every positive measure μ and for every μ-continuous measure ν of bounded variation with values in X, there exists an integrable function f with values in X so that

$$\nu(A) = \int_A f d\mu$$

for every measurable set A. The RNP can be characterized in terms of martingale convergence, i.e., for a Banach space X, all L^1-bounded martingales (f_n) with values in X converge almost surely if and only if X has the RNP.
In this talk, we give a similar characterization of the RNP in terms of polynomial spline sequences instead of martingales.

The asymptotic volume ratio of Schatten p-classes

Joscha Prochno, University of Hull

The Schatten trace classes S_p $(0 < p \leq \infty)$, consisting of all compact linear operators on a Hilbert space for which the sequence of their singular values belongs to the sequence space l_p, are one of the most important classes of unitary operator ideals. Their analysis, particularly in the finite-dimensional setting, has a long tradition in asymptotic geometric analysis and the local theory of Banach spaces. For example, Gordon and Lewis (1974) obtained that the space S_1 does not have local unconditional structure, Tomzcak-Jaegermann (1974) demonstrated that this space has Rademacher cotype 2, and Knig, Meyer and Pajor (1998) proved the boundedness of the isotropic constants of S_p^n $(1 \leq p \leq \infty)$. More recently, Guédon and Paouris (2007) have established concentration of mass properties for the unit balls of the Schatten p-classes S_p^n, Barthe and Cordero-Erausquin (2013) studied variance estimates, Radke and Vritsiou (2016) proved the thin-shell conjecture, and Hinrichs, Prochno and Vybíral (2017) computed the entropy numbers for their natural embeddings.

In [Studia Math. 80, 63–75, 1984], Saint Raymond studied the volumetric properties of unit balls in finite-dimensional real and complex Schatten p-classes and his results were used in most of the previously mentioned works. He obtained an asymptotic formlua for their volume, which contains an unknown limiting constant for which he provided both lower and upper bounds. We determine the exact limiting constant and as an application compute the precise asymptotic volume ratio of Schatten p-classes as the dimension tends to infinity. This extends Saint Raymond's estimate in the case of the nuclear norm (p=1) to the full regime $1 \leq p \leq \infty$ with exact limiting behavior.

(Joint work with Z. Kabluchko and C. Thäle)

Separated families of unit vectors

Tommaso Russo, Università degli Studi di Milano

Over the last years, a renewed interest and a rapid progress in delineating the structure of both qualitative and quantitative properties of well-separated subsets of the unit sphere of a Banach space have been observed. In particular, several results are available that give conditions on a non-separable Banach space for its unit sphere to contain an uncountable set whose distinct elements have distances strictly greater than 1, or even geater than $1 + \varepsilon$, for some uniform $\varepsilon > 0$.
In this talk, based on a joint work with Petr Hájek and Tomasz Kania, we shall give an introduction to the subject and present some recent results.

Gaps, additive energy and Poissonian pair correlation problems

Wolfgang Stockinger, Johannes Kepler University, Linz

We say that a sequence $(x_n)_{n\in\mathbb{N}}$ in $[0,1)$ has Poissonian pair correlations if

$$\lim_{N\to\infty}\frac{1}{N}\#\left\{1\le l\neq m\le N : \|x_l - x_m\|\le \frac{s}{N}\right\} = 2s$$

for every $s \geq 0$. We will discuss a gap theorem which allows to deduce that a sequence $(x_n)_{n\in\mathbb{N}}$ of real numbers in $[0,1)$ having a certain weak gap structure cannot have Poissonian pair correlations. This result covers a broad class of sequences, e.g., Kronecker sequences, the van der Corput sequence and in more general LS-sequences of points and digital $(t,1)$-sequences. Additionally, this theorem enables us to derive negative pair correlation properties for sequences of the form $(\{a_n\alpha\})_{n\in\mathbb{N}}$, where the sequence $(a_n)_{n\in\mathbb{N}}$ is quasi-arithmetic of degree 1. This statement is not only a metrical result, but holds for all possible choices of α.

A unified approach to decompose positive elements

Tamas Titkos, Alfréd Rényi Institute of Mathematics, Budapest

Several Lebesgue type decomposition theorems in functional analysis have a strong relation to the operation called: parallel addition. Using a natural generalization of Arlinskiis approach (which identifies the singular part as a fixed point of a single-variable map) we give a short proof for the existence of a Lebesgue type decomposition for positive operators, representable functionals, and nonnegative finite measures. The focus is on the fact that each theorem can be proved with the same completely elementary method.

The inverse of the dispersion depends logarithmically on the dimension

Mario Ullrich, Johannes Kepler University, Linz

We discuss recent developments on the dispersion of a point set, which is defined as the volume of the largest axis-parallel box in the unit cube that do not intersect the point set, with special emphasis on the dependence on the dimension.

Approximation of structured functions and simple neural networks

Jan Vybíral, Czech Technical University, Prague

The approximation of multivariate smooth functions is known to suffer by curse of dimension. Therefore, the approximation of functions with some special inner structure attracted recently a lot of attention. We review some of the recent results, including the approximation of ridge functions, of sparse additive models and of sums of ridge functions. The last class can be also interpreted as simple neural networks. It is a joint work with I. Daubechies (Duke), M. Fornassier (TU Munich), K. Schnass (Uni Innsbruck), and H. Tyagi (Alan Turing Institute).

KdV hierarchy via Abelian coverings and operator identities

Petro Yudytskiy, Johannes Kepler University, Linz

We establish precise spectral criteria for potential functions V of reflectionless Schrödinger operators $L_V = -\partial_x^2 + V(x)$ to admit solutions to the Korteweg de-Vries (KdV) hierarchy with V as an initial value. More generally, our methods extend the classical study of algebro-geometric solutions for the KdV hierarchy to noncompact Riemann surfaces by defining generalized Abelian integrals and analogues of the Baker-Akhiezer function on infinitely connected domains with a uniformly thick boundary satisfying a fractional moment condition.

Joint work with Benjamin Eichinger and Tom VandenBoom.

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