Vorlesung/Lectures by Alexander K. Belyaev

Modelling in the vibration theory

Modellbildung in der Technischen Schwingungslehre

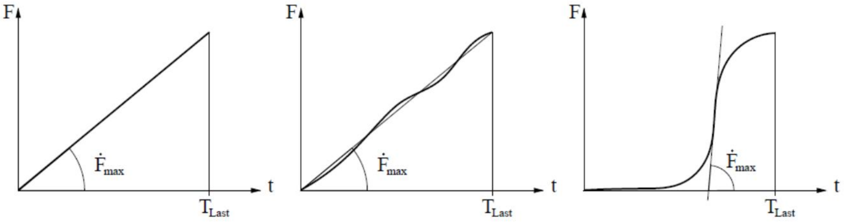
This course of lectures is important for the mechatronics curriculum for the following reasons. Roughly speaking, the mechatronics is known to be a branch of science that combines mechanics and control, as well electronics, computer science, telecommunication and many others. A correct mathematical model of the plant of the designed mechatronic system is of crucial importance for design of the controller. This model should be appropriate to capture the main features of the system to be controlled whereas all the features of “the second importance” should be ignored in order to keep the model as simple as possible. If the mathematical model of the mechanical system to be controlled is not able to capture its dynamic properties then the automatic control of this system is hardly feasible. Strictly speaking, the control is still possible however it is inefficient and energy consuming since this control does not fit the dynamics of the system. This also results in excessive forces and stresses in the mechanical systems which is always undesirable.

The content of the course is listed below with some short comments for each theme.

• **Forces in vibration theory (Kräfte in Schwingungslehre)** It addresses the main terminology of dynamics and vibration theory which is needed for proper understanding the mechanical terms.

• **Choice of the generalized coordinates and consequences of the incorrect choice** (**Auswahl der Lagekoordinaten und Konsequenzen der fehlerhaften Auswahl**) It concerns with the definition of generalized coordinates and contains the benchmark of angular and Cartesian coordinates. Merits and shortcomings of these types of coordinates are discussed. An example of inappropriate choice of coordinates is presented.

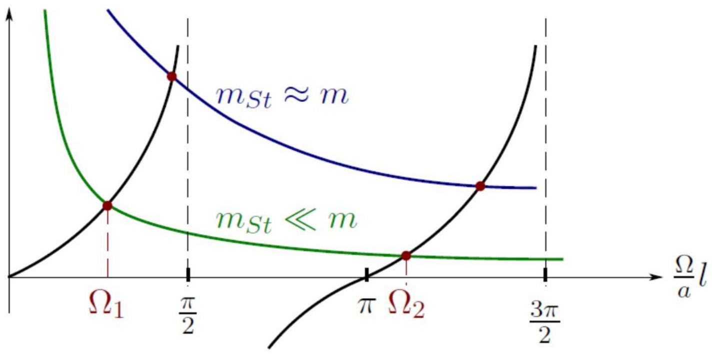
• Static and dynamic loads (Statische oder dynamische Last?) It contains some simple rules as to how to meet a decision whether a particular loading for some mechanical system can be treated as dynamic or static load. It allows one to decide what type of simulation (dynamic or static) is needed after a simple preliminary analysis. It also helps one to decide between dynamic or static model for plant of the control system.



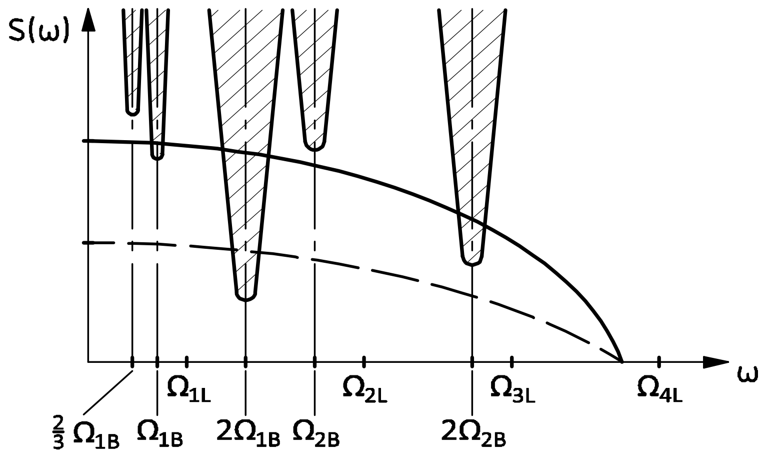

$$\frac{F}{\dot{F}_{max}} = T_{Last} \gg \frac{T}{2} \qquad T_{Last} \gg \frac{T}{2} \qquad \frac{F}{\dot{F}_{max}} \gg \frac{T}{2}$$

Some typical cases of loading

• **Discretisation of mechanical systems (Diskretisierung mechanischer Systeme)** It deals with the strategy of discretisation of distributed mechanical systems and resulted in the advices as to how many degrees of freedom is required for correct approximation of the distributed mechanical system for a given external load. The classification of mechanical and acoustical vibrations is given.

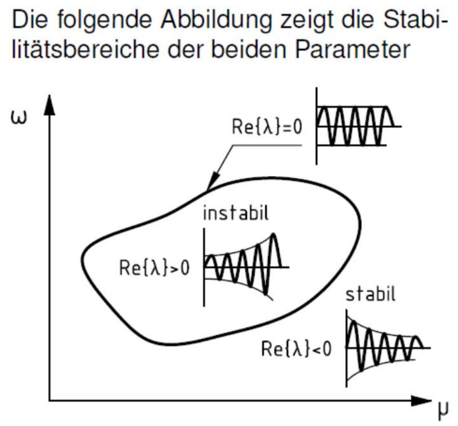


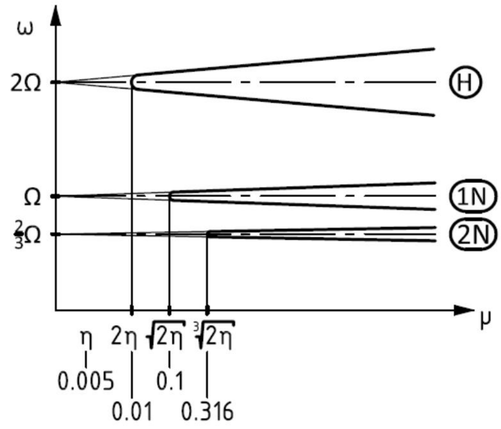

Eigenfrequencies of mechanical and acoustical vibrations




Spectrum of the external force and eigenfrequencies of axial and bending vibrations along with the stability chart

• **Instability of non-conservative dynamical systems (Instabilitaet bei nichtkonservativen Systemen)** To some extent, it is a certain continuation of the previous theme which demonstrates a necessity of introducing additional degrees of freedom for describing both static and dynamics instability in distributed systems.



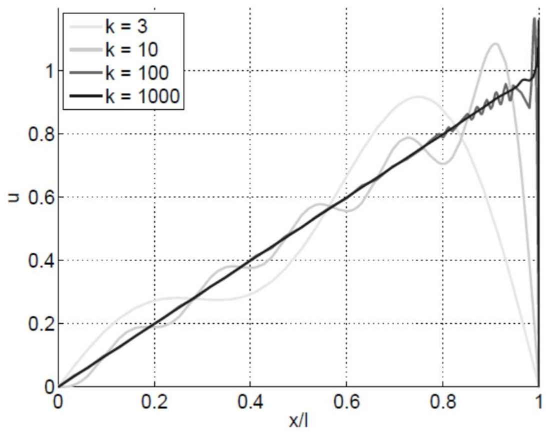




The boundary of instability region

Stability chart for a beam

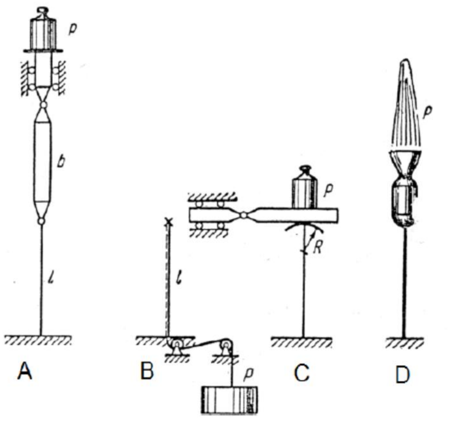
• **Application of the Hamilton variational principle for derivation of boundary-value problems (Anwendung des Variationprinzips von Hamilton fuer die Herleitung der Randwertprobleme)** Hamilton's variational principle is explained which allows one to obtain the boundary-value problems in terms of the kinetic and potential energy of mechanical system. A number of examples are provided. Strong and weak sides of the approach are discussed.

• **Modal analysis and improvement of convergence (Modale Analyse and Verbesserung der Konvergenz)** This standard approach of the vibration theory is explained in detail. It is demonstrated that a conventional application of the modal analysis yields the wrong values of displacement on the boundary of the body. A method of improvement of the convergence is suggested.

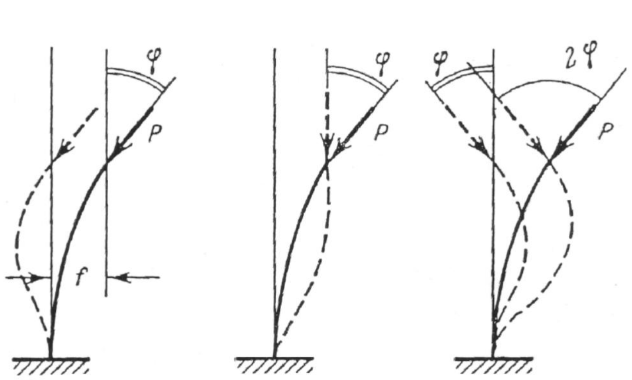



Bad convergence and Gibbs phenomenon as result of blind application of modal analysis

• **Sensitivity of the buckling load to the load application method (Empfindlichkeit der Knicklast gegenueber der Belastungsmethoden)** It is shown that the use of the formula for critical (Euler) load taken from the reference literature is misleading in number of cases. The critical load is demonstrated to be very sensitive to the particular way of loading. The accurate modeling shows that the actual critical loading can be greater or smaller than the Euler critical force. Special attention is paid to the case of follower load.

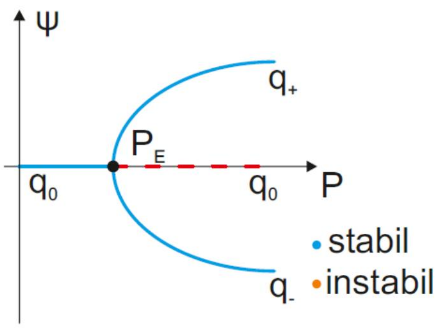



Variety of technical ways of loading

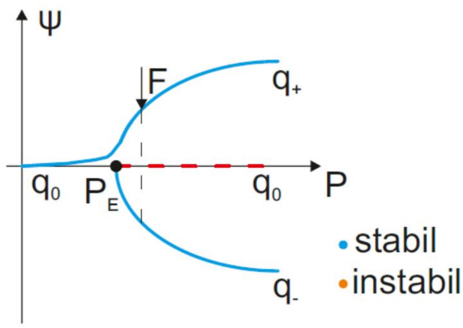


Follower force depends on the way of loading

• **Robustness of modeling (Robustheit der Modellbildung)** The concept of the structural instability is considered. It is shown that the mechanical parameters of the system can be conventionally divided into two groups. The first group contains those parameters to which the critical load is robust. In contrast, the critical load is so sensitive to parameters of the second group that some small changes in these parameters leads to qualitative changes in the equations and graphs for the critical load in the plane of parameters.

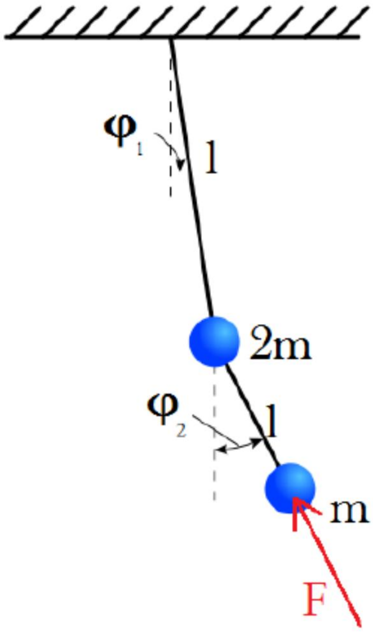



Perfect system

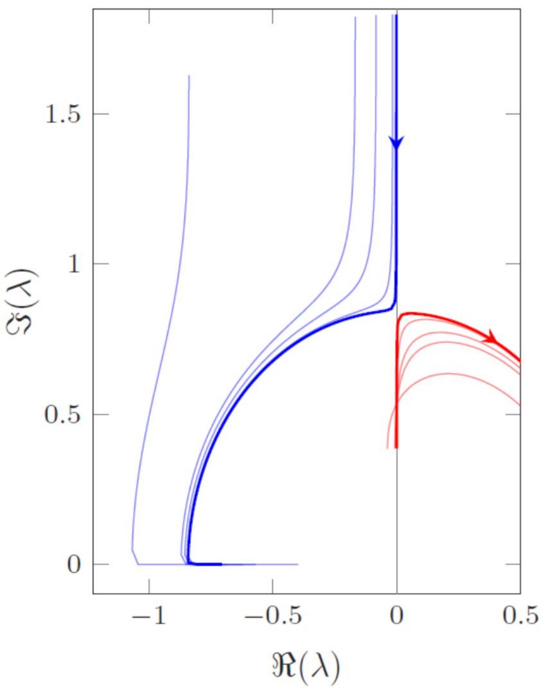



System with imperfection

• **Ziegler's paradox and search for critical parameters (Das Ziegler Paradoxon und Suche der kritischen Parameter)** This theme is devoted to the paradoxal result that the value of critical load depends upon whether i) zero damping is assumed or ii) non-zero damping is first introduced and then the damping vanishes. A discussion on the role of damping, types of instability and search for critical parameter is provided.




Double pendulum by Ziegler




Real part of eigenvalue for several parameters