A detailed description of each flipped learning experiment phase

Phase 1
The first phase of our flipped learning experiment (FLE) took place in two schools in May and June 2018 with both authors working as teachers and researchers. At the beginning of the learning sequences, all students were informed about the tasks to be performed, the goals to be achieved and the deadline. In the college of business administration, students had to solve different tasks, which were made available via an online learning platform. There were also short, interactive videos with integrated questions provided to check students' comprehension. Therefore, watching a video became an active process. In the high school, students had to research Leonardo bridges, build such a bridge themselves, and finally mathematically examine this self-built bridge. Students were able to use tablets (college of business administration), their notebooks (high school) and other learning materials according to their preferences.

After the first phase, we collected written feedback on the teaching and learning experiment. We opted for written feedback to be able to collect data from all students involved in our FLE. The feedback guidance was formulated openly and broadly, as on the one hand, new hypotheses and theories were to be developed based on feedback forms. On the other hand, precise guiding questions have been omitted in order to reduce the risk of students answering as they think teachers would like them to. The written feedback from the students and the teaching notes of the authors were the basis for the first scientific investigation and evaluation of our FLE. Based on the first scientific evaluations, we deduced preliminary results of the teaching and learning experiment and the instructional design was modified for the second phase.

Phase 2
The second phase of our FLE took place between September and October 2018. In order to gain both a scientific internal and an external perspective on our FLE, we decided that one author should continue to be active as a researcher and teacher, but the second author should only take on the role of a not-involved researcher. This division of perspectives on flipped learning mathematics education means that from September 2018 on, our teaching experiment was carried out only at the grammar school with two classes. From the second phase onwards, two classes (9th and 10th grade) with slightly less than 50 students were involved in our FLE.

In both classes, the ePortfolio software Mahara was used as a communication and presentation tool. The teacher had an ePortfolio-page for each class where the teacher communicated and provided work assignments as well as learning materials. Also, all students had a Mahara-pages where students documented their progress, presented learning artefacts and shared them with the teacher and classmates. The sharing of learning artefacts took place in a secure online space, and only classmates and the teacher had access to this secure online space.

The learning and working process in the second phase was very similar in both classes involved: At the beginning of the learning sequence, we provided the pupils work assignments, learning materials and we communicated the end of the learning sequence. After communicating and making available the work assignments and learning resources, the pupils were able to work in the classroom and at home on achieving the learning objectives. The teacher was available to answer questions and assist with this process both in class and online. At the end of the learning sequence, students had to upload theoretical and practical learning products to their Mahara-page and share the page with the teacher and classmates.

Phase 3
The third phase of the teaching and learning experiment took place from November to December 2018, and the design was based on the findings and experiences of the second phase. A noticeable change in the third phase design was that there was a more detailed subdivision of the flipped learning sequence. In the first sequence, the students had to deal

with general and theoretical questions and tasks on a new mathematical topic. Those aspects or theoretical parts of the new mathematical topic that caused problems or ambiguities for the students were discussed in class in on-demand lectures at the end of the theoretical part. After discussing the theoretical aspects of mathematical concepts, we provided the students with work assignments and additional learning aids. Both the theoretical and the practical part of our FLE consisted of two sub-phases. After each sub-phase, the mathematical concepts treated and worked on were discussed, and students could address any ambiguities so that any misconceptions could be identified and solved as early as possible. At the end of the second practical learning phase, all students had to finish their Mahara-page on the new mathematical topic and release it.

Also new in this phase was that at regular intervals, a teacher who was not involved in the research process was invited to observe the lessons. The observing teacher taught one class of our FLE in physics and provided us with feedback on lesson planning and lesson design, learning actions of the students as well as activities and roles of the teacher after the lesson observations.

Phase 4

In January and February 2019, we conducted the fourth phase of the FLE with a new design. One difference in the fourth phase was that there was a stronger structuring of the lessons. Stronger structuring means that the students worked on the theory and practical applications of new mathematical concepts in one or two weekly plans.

Also, in addition to input and explanations in the lessons, students were provided with online meta-information on using technologies (Mahara and GeoGebra).

A further innovation of this phase was that lessons in the computer lab and a traditional classroom were offered at regular intervals. With this division of the learning environment, one teacher was available to the students in each environment so that the students could ask questions in each learning environment. The choice of the learning environment was left to the students during this phase, and they could also change the learning environment during the lesson if they felt it was appropriate to do so.

Interviews were conducted with the students during and after this phase. The interviews focused on the students' wishes, needs and problems regarding learning in the new learning environment. We conducted individual as well as group interviews in a calm environment in the school. While and after the last phase of our educational experiment, we collected written feedback from the students regarding the changes in the last phase.