4^{th} practice sheet multivariate methods II

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13. In order to get a unique loading matrix $\mathbf{L}$ for the fundamental theorem $\mathbf{\Sigma} = \mathbf{L}\mathbf{L}^T + \mathbf{V}$ with given k and $\mathbf{V}$ we might use the side condition $\mathbf{L}^T\mathbf{V}^{-1}\mathbf{L} =$ diagonal.
Show: equivalent to this condition is the following: determine $\mathbf{L}$ as with principal axis transformation of $\mathbf{V}^{-\frac{1}{2}}(\mathbf{\Sigma} - \mathbf{V})\mathbf{V}^{-\frac{1}{2}}$ and scale the result such that it satisfies the fundamental theorem for $\mathbf{\Sigma}$.

14. Given is the model of ML-factor analysis. Show: with Bartlett's test (test, whether the number of factors $k = 0$) we may substitute $-\ln|\mathbf{R}|$ for the term $(\ln|\hat{\mathbf{\Sigma}}| - \ln|\mathbf{S}|)$ in the testing statistics U_0 ($\mathbf{R}$ is the empirical correlation matrix).

15. 114 students of psychology were asked the reason why they chose their field of study. There have been 13 reasons, the students had to evaluate on a 7-point scale how these reasons influenced their choice.

    To structure the 13 reasons a factor analysis was performed. A smaller number of separable groups of reasons should be found. From the data we get the following empirical correlation matrix (You find the correlation matrix in `correl.sd2`):

$$\mathbf{R} = \begin{pmatrix}
1 & -.02 & .091 & .145 & .145 & -.122 & .152 & -.003 & .048 & .091 & .095 & .141 & -.074 \\
 & 1 & -.11 & .393 & .459 & -.023 & -.076 & .375 & .025 & -.072 & .208 & .267 & .072 \\
 & & 1 & .043 & -.134 & .149 & .696 & -.085 & .063 & .102 & -.027 & -.025 & -.021 \\
 & & & 1 & .316 & -.077 & .049 & .296 & .121 & -.071 & .38 & .003 & -.029 \\
 & & & & 1 & .063 & .045 & .406 & .216 & .077 & .312 & .298 & .025 \\
 & & & & & 1 & .238 & -.018 & .475 & .297 & .215 & .316 & .124 \\
 & & & & & & 1 & -.094 & .24 & .233 & .218 & .111 & -.01 \\
 & & & & & & & 1 & .05 & -.03 & .154 & .135 & .031 \\
 & & & & & & & & 1 & .38 & .374 & .284 & .113 \\
 & & & & & & & & & 1 & .17 & .164 & -.001 \\
 & & & & & & & & & & 1 & .349 & .001 \\
 & & & & & & & & & & & 1 & .099 \\
 & & & & & & & & & & & & 1
\end{pmatrix}$$

    Perform a ML-factor analysis with the SAS-procedure FACTOR and determine the number of factors with Bartlett's likelihood-ratio-test.

16. Let $\mathbf{x} = \binom{x_1}{x_2}$ be normally distributed $\mathbf{N}(\mathbf{o}; \mathbf{\Sigma})$ with $\mathbf{\Sigma} = \begin{pmatrix} 1 & 1/2 \\ 1/2 & 1 \end{pmatrix}$. Compute the principal axes $\mathbf{h} = \binom{h_1}{h_2}$ of $\mathbf{x}$. What is the probability distribution of $\mathbf{h}$?