Multivariate Verfahren 2

factor analysis

Helmut Waldl

May 14th and 15th 2012

factor analysis

factor transformation

Sometimes the factors computed with ML factor analysis or principal component analysis can hardly be interpreted.

After nonsingular linear transformations the factors may often be interpreted better.

Orthogonal transformations are linear and nonsingular, orthonormal factors remain orthonormal after the transformation.

Non-orthogonal (oblique) transformations allow for dependent factors.

Less the product of loadings and factors and their contribution to the total variance of the variables change, we have to apply the inverse transformation to the matrix of loadings.

factor analysis

factor transformation

Sometimes the factors computed with ML factor analysis or principal component analysis can hardly be interpreted.

After nonsingular linear transformations the factors may often be interpreted better.

Orthogonal transformations are linear and nonsingular, orthonormal factors remain orthonormal after the transformation.

Non-orthogonal (oblique) transformations allow for dependent factors.

Less the product of loadings and factors and their contribution to the total variance of the variables change, we have to apply the inverse transformation to the matrix of loadings.

factor analysis

factor transformation

Sometimes the factors computed with ML factor analysis or principal component analysis can hardly be interpreted.

After nonsingular linear transformations the factors may often be interpreted better.

Orthogonal transformations are linear and nonsingular, orthonormal factors remain orthonormal after the transformation.

Non-orthogonal (oblique) transformations allow for dependent factors.

Less the product of loadings and factors and their contribution to the total variance of the variables change, we have to apply the inverse transformation to the matrix of loadings.

factor analysis

factor transformation

Sometimes the factors computed with ML factor analysis or principal component analysis can hardly be interpreted.

After nonsingular linear transformations the factors may often be interpreted better.

Orthogonal transformations are linear and nonsingular, orthonormal factors remain orthonormal after the transformation.

Non-orthogonal (oblique) transformations allow for dependent factors.

Less the product of loadings and factors and their contribution to the total variance of the variables change, we have to apply the inverse transformation to the matrix of loadings.

factor analysis

factor transformation

Sometimes the factors computed with ML factor analysis or principal component analysis can hardly be interpreted.

After nonsingular linear transformations the factors may often be interpreted better.

Orthogonal transformations are linear and nonsingular, orthonormal factors remain orthonormal after the transformation.

Non-orthogonal (oblique) transformations allow for dependent factors.

Less the product of loadings and factors and their contribution to the total variance of the variables change, we have to apply the inverse transformation to the matrix of loadings.

factor analysis

factor transformation M

Let M be a nonsingular $(k \times k)$-matrix

$$f \mapsto M^{-1}f = \tilde{f} \qquad L \mapsto L \cdot M = \tilde{L}$$

$\Longrightarrow L \cdot f = L \cdot M \cdot M^{-1}f = \tilde{L} \cdot \tilde{f}$ remains unchanged in the partitioning of $\mathbf{y} - \mu = L \cdot f + e$.

For the covariance matrix we get

$$\Sigma = Cov(L \cdot f) + Cov(e) = L \cdot L^T + V =$$

$$= Cov(\tilde{L} \cdot \tilde{f}) + Cov(e) = \tilde{L} \cdot \Phi \cdot \tilde{L}^T + V$$

with $\Phi = Cov(\tilde{f}) = M^{-1}M^{-T}$

Φ is the covariance matrix of the transformed (rotated) factors. There exists a great many of criteria to choose a proper transformation or rotation matrix M.

factor analysis

factor transformation M

Let M be a nonsingular $(k \times k)$-matrix

$$f \mapsto M^{-1}f = \tilde{f} \qquad L \mapsto L \cdot M = \tilde{L}$$

$\Longrightarrow L \cdot f = L \cdot M \cdot M^{-1} f = \tilde{L} \cdot \tilde{f}$ remains unchanged in the partitioning of $\mathbf{y} - \boldsymbol{\mu} = L \cdot f + e$.

For the covariance matrix we get

$$\Sigma = Cov(L \cdot f) + Cov(e) = L \cdot L^T + V =$$

$$= Cov(\tilde{L} \cdot \tilde{f}) + Cov(e) = \tilde{L} \cdot \Phi \cdot \tilde{L}^T + V$$

with $\Phi = Cov(\tilde{f}) = M^{-1}M^{-T}$

Φ is the covariance matrix of the transformed (rotated) factors. There exists a great many of criteria to choose a proper transformation or rotation matrix M.

factor analysis

factor transformation M

Let M be a nonsingular $(k \times k)$-matrix

$$f \mapsto M^{-1}f = \tilde{f} \qquad L \mapsto L \cdot M = \tilde{L}$$

$\Longrightarrow L \cdot f = L \cdot M \cdot M^{-1}f = \tilde{L} \cdot \tilde{f}$ remains unchanged in the partitioning of $\mathbf{y} - \boldsymbol{\mu} = L \cdot f + e$.

For the covariance matrix we get

$$\Sigma = Cov(L \cdot f) + Cov(e) = L \cdot L^T + V =$$

$$= Cov(\tilde{L} \cdot \tilde{f}) + Cov(e) = \tilde{L} \cdot \Phi \cdot \tilde{L}^T + V$$

with $\Phi = Cov(\tilde{f}) = M^{-1}M^{-T}$

Φ is the covariance matrix of the transformed (rotated) factors. There exists a great many of criteria to choose a proper transformation or rotation matrix M.

factor analysis

factor transformation M

Let M be a nonsingular $(k \times k)$-matrix

$$f \mapsto M^{-1}f = \tilde{f} \qquad L \mapsto L \cdot M = \tilde{L}$$

$\Longrightarrow L \cdot f = L \cdot M \cdot M^{-1}f = \tilde{L} \cdot \tilde{f}$ remains unchanged in the partitioning of $\mathbf{y} - \boldsymbol{\mu} = L \cdot f + e$.

For the covariance matrix we get

$$\Sigma = Cov(L \cdot f) + Cov(e) = L \cdot L^T + V =$$

$$= Cov(\tilde{L} \cdot \tilde{f}) + Cov(e) = \tilde{L} \cdot \Phi \cdot \tilde{L}^T + V$$

with $\Phi = Cov(\tilde{f}) = M^{-1}M^{-T}$

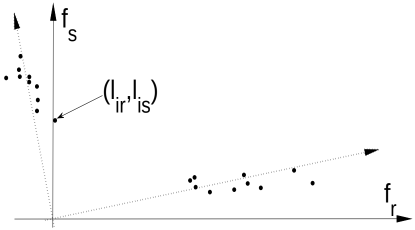
Φ is the covariance matrix of the transformed (rotated) factors. There exists a great many of criteria to choose a proper transformation or rotation matrix M.

factor analysis

rotation matrices, factor rotation

We start with the factor analysis model $\mathbf{y} - \boldsymbol{\mu} = L \cdot \mathbf{f} + \mathbf{e}$. If we neglect the unique factors $\mathbf{e}$ we could say:

The original variables are put into a coordinate system spanned by the normalized factors, the loadings $(l_{i1}, \ldots, l_{ik})$, $i = 1, \ldots, p$ are the coordinates of the variables: $L \cdot \mathbf{f}$



each point stands for one variable

For visualization purposes we project the variables on two orthogonal factors f_r and f_s.

The factors would describe the variables better if the factor axes would go through the clusters in the scatter plot.

Solution:

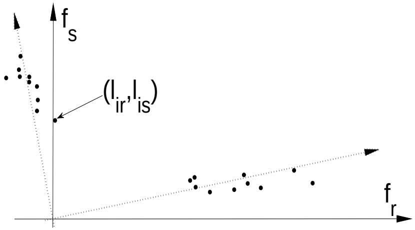
We rotate the coordinate axes.

factor analysis

rotation matrices, factor rotation

We start with the factor analysis model $\mathbf{y} - \boldsymbol{\mu} = L \cdot \mathbf{f} + \mathbf{e}$. If we neglect the unique factors $\mathbf{e}$ we could say:

The original variables are put into a coordinate system spanned by the normalized factors, the loadings $(l_{i1}, \ldots, l_{ik}),\ i = 1, \ldots, p$ are the coordinates of the variables: $L \cdot \mathbf{f}$



each point stands for one variable

For visualization purposes we project the variables on two orthogonal factors f_r and f_s.

The factors would describe the variables better if the factor axes would go through the clusters in the scatter plot.

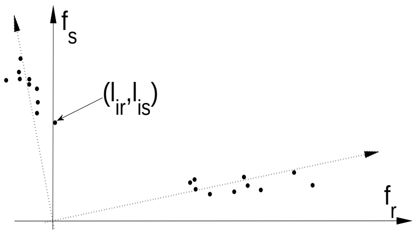
Solution:
We rotate the coordinate axes.

factor analysis

rotation matrices, factor rotation

We start with the factor analysis model $\mathbf{y} - \boldsymbol{\mu} = L \cdot \mathbf{f} + \mathbf{e}$. If we neglect the unique factors $\mathbf{e}$ we could say:

The original variables are put into a coordinate system spanned by the normalized factors, the loadings $(l_{i1}, \ldots, l_{ik}),\ i = 1, \ldots, p$ are the coordinates of the variables: $L \cdot \mathbf{f}$



each point stands for one variable

For visualization purposes we project the variables on two orthogonal factors f_r and f_s.

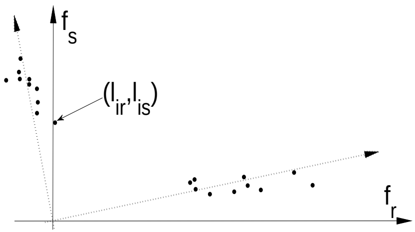
The factors would describe the variables better if the factor axes would go through the clusters in the scatter plot.

Solution:

We rotate the coordinate axes.

We start with the factor analysis model $\mathbf{y} - \boldsymbol{\mu} = L \cdot \mathbf{f} + \mathbf{e}$. If we neglect the unique factors $\mathbf{e}$ we could say:

The original variables are put into a coordinate system spanned by the normalized factors, the loadings $(l_{i1}, \ldots, l_{ik}),\ i = 1, \ldots, p$ are the coordinates of the variables: $L \cdot \mathbf{f}$



each point stands for one variable

For visualization purposes we project the variables on two orthogonal factors f_r and f_s.

The factors would describe the variables better if the factor axes would go through the clusters in the scatter plot.

Solution:

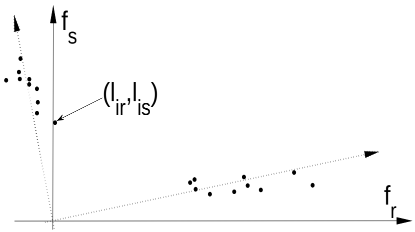
We rotate the coordinate axes.

factor analysis

rotation matrices, factor rotation

We start with the factor analysis model $\mathbf{y} - \boldsymbol{\mu} = L \cdot \mathbf{f} + \mathbf{e}$. If we neglect the unique factors $\mathbf{e}$ we could say:

The original variables are put into a coordinate system spanned by the normalized factors, the loadings $(l_{i1}, \ldots, l_{ik}),\ i = 1, \ldots, p$ are the coordinates of the variables: $L \cdot \mathbf{f}$



each point stands for one variable

For visualization purposes we project the variables on two orthogonal factors f_r and f_s.

The factors would describe the variables better if the factor axes would go through the clusters in the scatter plot.

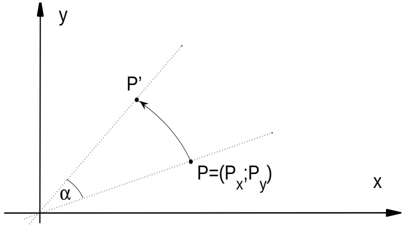
Solution:
We rotate the coordinate axes.

factor analysis

rotation matrices, 2D rotation

We rotate of a point P counterclockwise about zero in a two-dimensional coordinate system

$$\begin{pmatrix} P'_x \\ P'_y \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \cdot \begin{pmatrix} P_x \\ P_y \end{pmatrix}$$



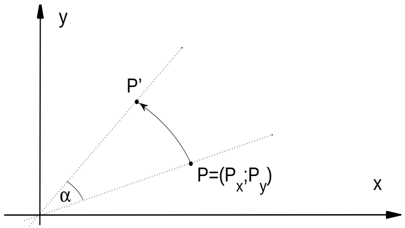
We get the same result if we leave P unchanged and rotate the coordinate axes instead in the reverse direction.

factor analysis

rotation matrices, 2D rotation

We rotate of a point P counterclockwise about zero in a two-dimensional coordinate system

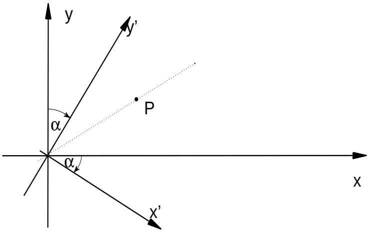
$$\begin{pmatrix} P'_x \\ P'_y \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \cdot \begin{pmatrix} P_x \\ P_y \end{pmatrix}$$



We get the same result if we leave P unchanged and rotate the coordinate axes instead in the reverse direction.

factor analysis

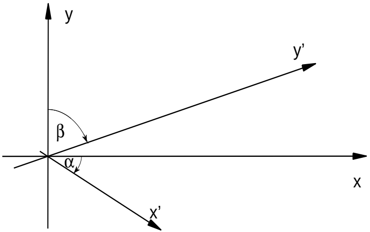
rotation matrices, 2D rotation



In the (x, y)-coordinate system P' is located just as P is located in the (x', y')-coordinate system.

New coordinate system:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

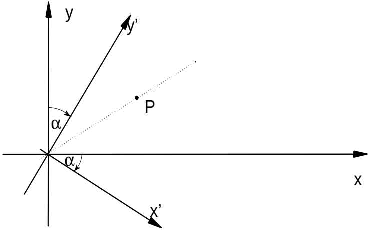


We may rotate the coordinate axes also with different angles α and β and get an oblique coordinate system.

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\beta \\ \sin\alpha & \cos\beta \end{pmatrix}^{-1} \begin{pmatrix} x \\ y \end{pmatrix}$$

factor analysis

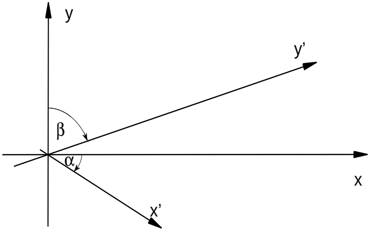
rotation matrices, 2D rotation



In the (x, y)-coordinate system P' is located just as P is located in the (x', y')-coordinate system.

New coordinate system:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

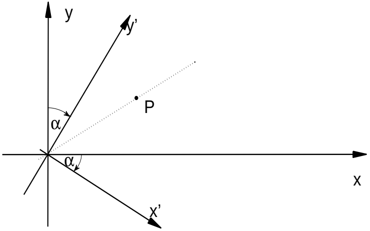


We may rotate the coordinate axes also with different angles α and β and get an oblique coordinate system.

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\beta \\ \sin\alpha & \cos\beta \end{pmatrix}^{-1} \begin{pmatrix} x \\ y \end{pmatrix}$$

factor analysis

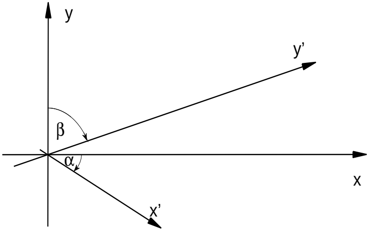
rotation matrices, 2D rotation



In the (x, y)-coordinate system P' is located just as P is located in the (x', y')-coordinate system.

New coordinate system:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

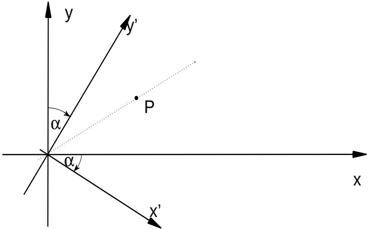


We may rotate the coordinate axes also with different angles α and β and get an oblique coordinate system.

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\beta \\ \sin\alpha & \cos\beta \end{pmatrix}^{-1} \begin{pmatrix} x \\ y \end{pmatrix}$$

factor analysis

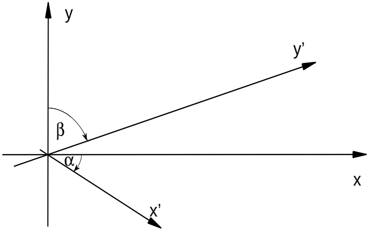
rotation matrices, 2D rotation



In the (x, y)-coordinate system P' is located just as P is located in the (x', y')-coordinate system.

New coordinate system:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$



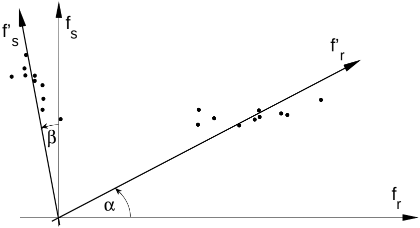
We may rotate the coordinate axes also with different angles α and β and get an oblique coordinate system.

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\beta \\ \sin\alpha & \cos\beta \end{pmatrix}^{-1} \begin{pmatrix} x \\ y \end{pmatrix}$$

factor analysis

rotation matrices, factor rotation

Back to our problem: We rotate the factors such that they are located "as close as possible" to the variables.



$$\begin{pmatrix} \tilde{f}_r \\ \tilde{f}_s \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\alpha & -\sin\beta \\ \sin\alpha & \cos\beta \end{pmatrix}}_{M^{-1}} \begin{pmatrix} f_r \\ f_s \end{pmatrix}$$

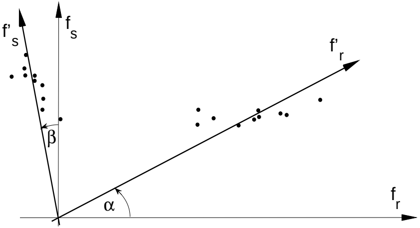
In order that $L \cdot f = \tilde{L} \cdot \tilde{f}$ we have to apply the inverse transformation to L:

$$\tilde{L} = L \cdot M = L \cdot \begin{pmatrix} \cos\alpha & -\sin\beta \\ \sin\alpha & \cos\beta \end{pmatrix}^{-1}$$

factor analysis

rotation matrices, factor rotation

Back to our problem: We rotate the factors such that they are located "as close as possible" to the variables.



$$\begin{pmatrix} \tilde{f}_r \\ \tilde{f}_s \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\alpha & -\sin\beta \\ \sin\alpha & \cos\beta \end{pmatrix}}_{M^{-1}} \begin{pmatrix} f_r \\ f_s \end{pmatrix}$$

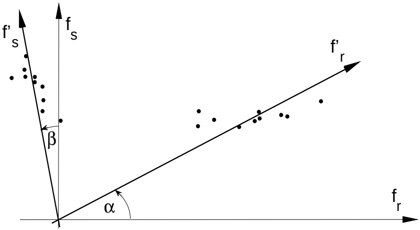
In order that $L \cdot f = \tilde{L} \cdot \tilde{f}$ we have to apply the inverse transformation to L:

$$\tilde{L} = L \cdot M = L \cdot \begin{pmatrix} \cos\alpha & -\sin\beta \\ \sin\alpha & \cos\beta \end{pmatrix}^{-1}$$

factor analysis

rotation matrices, factor rotation

Back to our problem: We rotate the factors such that they are located "as close as possible" to the variables.



$$\begin{pmatrix} \tilde{f}_r \\ \tilde{f}_s \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\alpha & -\sin\beta \\ \sin\alpha & \cos\beta \end{pmatrix}}_{M^{-1}} \begin{pmatrix} f_r \\ f_s \end{pmatrix}$$

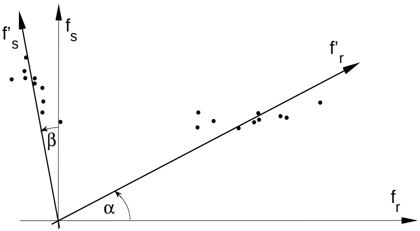
In order that $L \cdot f = \tilde{L} \cdot \tilde{f}$ we have to apply the inverse transformation to L:

$$\tilde{L} = L \cdot M = L \cdot \begin{pmatrix} \cos\alpha & -\sin\beta \\ \sin\alpha & \cos\beta \end{pmatrix}^{-1}$$

factor analysis

rotation matrices, factor rotation

Back to our problem: We rotate the factors such that they are located "as close as possible" to the variables.



$$\begin{pmatrix} \tilde{f}_r \\ \tilde{f}_s \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\alpha & -\sin\beta \\ \sin\alpha & \cos\beta \end{pmatrix}}_{M^{-1}} \begin{pmatrix} f_r \\ f_s \end{pmatrix}$$

In order that $L \cdot f = \tilde{L} \cdot \tilde{f}$ we have to apply the inverse transformation to L:

$$\tilde{L} = L \cdot M = L \cdot \begin{pmatrix} \cos\alpha & -\sin\beta \\ \sin\alpha & \cos\beta \end{pmatrix}^{-1}$$

factor analysis

rotation matrices, factor rotation

The described factor transformation of two factors is orthogonal iff $\alpha = \beta$ else we have an oblique transformation.

We may rotate all factors $f_1, \ldots, f_k$ to an arbitrary position with a sequence of the above 2D rotations, e.g.

$$M = \begin{pmatrix} \cos\alpha & -\sin\beta & 0 \\ \sin\alpha & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\gamma & 0 & -\sin\delta \\ 0 & 1 & 0 \\ \sin\gamma & 0 & \cos\delta \end{pmatrix}$$

It only remains unsettled how to determine the angles such that the factors are located "as close as possible" to the variables.

There are many disparate methods.

factor analysis

rotation matrices, factor rotation

The described factor transformation of two factors is orthogonal iff $\alpha = \beta$ else we have an oblique transformation.

We may rotate all factors $f_1, \ldots, f_k$ to an arbitrary position with a sequence of the above 2D rotations, e.g.

$$M = \begin{pmatrix} \cos\alpha & -\sin\beta & 0 \\ \sin\alpha & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\gamma & 0 & -\sin\delta \\ 0 & 1 & 0 \\ \sin\gamma & 0 & \cos\delta \end{pmatrix}$$

It only remains unsettled how to determine the angles such that the factors are located "as close as possible" to the variables.

There are many disparate methods.

factor analysis

rotation matrices, factor rotation

The described factor transformation of two factors is orthogonal iff $\alpha = \beta$ else we have an oblique transformation.

We may rotate all factors $f_1, \ldots, f_k$ to an arbitrary position with a sequence of the above 2D rotations, e.g.

$$M = \begin{pmatrix} \cos\alpha & -\sin\beta & 0 \\ \sin\alpha & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\gamma & 0 & -\sin\delta \\ 0 & 1 & 0 \\ \sin\gamma & 0 & \cos\delta \end{pmatrix}$$

It only remains unsettled how to determine the angles such that the factors are located "as close as possible" to the variables.

There are many disparate methods.

factor analysis

rotation matrices, factor rotation

The described factor transformation of two factors is orthogonal iff $\alpha = \beta$ else we have an oblique transformation.

We may rotate all factors $f_1, \ldots, f_k$ to an arbitrary position with a sequence of the above 2D rotations, e.g.

$$M = \begin{pmatrix} \cos\alpha & -\sin\beta & 0 \\ \sin\alpha & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\gamma & 0 & -\sin\delta \\ 0 & 1 & 0 \\ \sin\gamma & 0 & \cos\delta \end{pmatrix}$$

It only remains unsettled how to determine the angles such that the factors are located "as close as possible" to the variables.

There are many disparate methods.

factor analysis

rotation matrices, factor rotation

The described factor transformation of two factors is orthogonal iff $\alpha = \beta$ else we have an oblique transformation.

We may rotate all factors $f_1, \ldots, f_k$ to an arbitrary position with a sequence of the above 2D rotations, e.g.

$$M = \begin{pmatrix} \cos\alpha & -\sin\beta & 0 \\ \sin\alpha & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\gamma & 0 & -\sin\delta \\ 0 & 1 & 0 \\ \sin\gamma & 0 & \cos\delta \end{pmatrix}$$

It only remains unsettled how to determine the angles such that the factors are located "as close as possible" to the variables.

There are many disparate methods.

factor analysis

rotation methods

Varimax-method

After the rotation the loadings of some variables on a factor should be very high, the loadings of the remaining variables should be as small as possible.

The squared loadings should be either very small or very large, the **variance of the squared loadings** should be maximized:

Let $\tilde{L} = \left(\tilde{l}_{ij} \right)_{\substack{i=1\cdots p \\ j=1\cdots k}}$ $\quad \tilde{L} = L \cdot M$

Problem: find M such that M is orthogonal and

$$e_1 = \sum_{j=1}^{k} \sum_{i=1}^{p} (\tilde{l}_{ij}^2 - \overline{\tilde{l}_{+j}^2}) \longrightarrow \max!$$

Varimax-method

After the rotation the loadings of some variables on a factor should be very high, the loadings of the remaining variables should be as small as possible.

The squared loadings should be either very small or very large, the **variance of the squared loadings** should be maximized:

Let $\tilde{L} = \left(\tilde{l}_{ij} \right)_{\substack{i=1\cdots p \\ j=1\cdots k}}$ $\quad \tilde{L} = L \cdot M$

Problem: find M such that M is orthogonal and

$$e_1 = \sum_{j=1}^{k} \sum_{i=1}^{p} (\tilde{l}_{ij}^2 - \overline{\tilde{l}_{+j}^2}) \longrightarrow \max!$$

factor analysis

rotation methods

Varimax-method

After the rotation the loadings of some variables on a factor should be very high, the loadings of the remaining variables should be as small as possible.

The squared loadings should be either very small or very large, the **variance of the squared loadings** should be maximized:

Let $\tilde{L} = \left(\tilde{l}_{ij} \right)_{\substack{i=1\cdots p \\ j=1\cdots k}}$ $\quad \tilde{L} = L \cdot M$

Problem: find M such that M is orthogonal and

$$e_1 = \sum_{j=1}^{k} \sum_{i=1}^{p} (\tilde{l}_{ij}^2 - \overline{\tilde{l}_{+j}^2}) \longrightarrow \text{max}\,!$$

factor analysis

rotation methods

Varimax-method

After the rotation the loadings of some variables on a factor should be very high, the loadings of the remaining variables should be as small as possible.

The squared loadings should be either very small or very large, the **variance of the squared loadings** should be maximized:

Let $\tilde{L} = \left(\ \tilde{l}_{ij}\ \right)_{\substack{i=1\cdots p \\ j=1\cdots k}}$ $\quad \tilde{L} = L \cdot M$

Problem: find M such that M is orthogonal and

$$e_1 = \sum_{j=1}^{k} \sum_{i=1}^{p} (\tilde{l}_{ij}^2 - \overline{\tilde{l}_{+j}^2}) \longrightarrow \text{max!}$$

factor analysis

rotation methods

Varimax-method

After the rotation the loadings of some variables on a factor should be very high, the loadings of the remaining variables should be as small as possible.

The squared loadings should be either very small or very large, the **variance of the squared loadings** should be maximized:

Let $\tilde{L} = \left(\tilde{l}_{ij} \right)_{\substack{i=1\cdots p \\ j=1\cdots k}}$ $\quad \tilde{L} = L \cdot M$

<u>Problem:</u> find M such that M is orthogonal and

$$e_1 = \sum_{j=1}^{k} \sum_{i=1}^{p} (\tilde{l}_{ij}^2 - \overline{\tilde{l}_{+j}^2}) \longrightarrow \max!$$

factor analysis

rotation methods

Quartimax-method

Similar to Varimax, we just try to maximize the variance of the loadings within the rows instead within the columns (Varimax) of the loading matrix.

Orthomax-method

A combination of Varimax and Quartimax.

factor analysis

rotation methods

Quartimax-method

Similar to Varimax, we just try to maximize the variance of the loadings within the rows instead within the columns (Varimax) of the loading matrix.

Orthomax-method

A combination of Varimax and Quartimax.

factor analysis

rotation methods

Quartimax-method

Similar to Varimax, we just try to maximize the variance of the loadings within the rows instead within the columns (Varimax) of the loading matrix.

Orthomax-method

A combination of Varimax and Quartimax.

factor analysis

rotation methods

Quartimax-method

Similar to Varimax, we just try to maximize the variance of the loadings within the rows instead within the columns (Varimax) of the loading matrix.

Orthomax-method

A combination of Varimax and Quartimax.

factor analysis

rotation methods

Promax-method

Improves the result of the orthogonal Varimax rotation $\tilde{L} = L \cdot M$ with an oblique transformation $\tilde{\tilde{L}} = \tilde{L} \cdot \tilde{M}$

The loadings should be even closer to 0 or 1 than with Varimax:

Let $Q = (q_{ij}) = (q_1 \cdots q_k)$ with
$q_{ij} = |\tilde{l}_{ij}^{m-1}| \cdot \tilde{l}_{ij};\ i = 1, \ldots, p;\ j = 1, \ldots, k;\ 1 < m \leq 4$ (usually).
Let further $U = (u_1 \cdots u_k) = (\tilde{L}^T \cdot \tilde{L})^{-1} \tilde{L}^T Q$ (coefficients of the regression of the columns $q_1, \ldots, q_k$ on $\tilde{L}$).

The Promax transformation matrix is then given by

$$\tilde{M} = U \cdot D \qquad \text{where} \qquad D^2 = \text{diag}\{(U^T U)^{-1}\}$$

$\tilde{M}$ is U with normalized columns.

factor analysis

rotation methods

Promax-method

Improves the result of the orthogonal Varimax rotation $\tilde{L} = L \cdot M$ with an oblique transformation $\tilde{\tilde{L}} = \tilde{L} \cdot \tilde{M}$

The loadings should be even closer to 0 or 1 than with Varimax:

Let $Q = (q_{ij}) = (q_1 \cdots q_k)$ with
$q_{ij} = |\tilde{l}_{ij}^{m-1}| \cdot \tilde{l}_{ij};\ i = 1, \ldots, p;\ j = 1, \ldots, k;\ 1 < m \leq 4$ (usually).
Let further $U = (u_1 \cdots u_k) = (\tilde{L}^T \cdot \tilde{L})^{-1} \tilde{L}^T Q$ (coefficients of the regression of the columns $q_1, \ldots, q_k$ on $\tilde{L}$).

The Promax transformation matrix is then given by

$$\tilde{M} = U \cdot D \qquad \text{where} \qquad D^2 = \text{diag}\{(U^T U)^{-1}\}$$

$\tilde{M}$ is U with normalized columns.

factor analysis

rotation methods

Promax-method

Improves the result of the orthogonal Varimax rotation $\tilde{L} = L \cdot M$ with an oblique transformation $\tilde{\tilde{L}} = \tilde{L} \cdot \tilde{M}$

The loadings should be even closer to 0 or 1 than with Varimax:

Let $Q = (q_{ij}) = (q_1 \cdots q_k)$ with
$q_{ij} = |\tilde{l}_{ij}^{m-1}| \cdot \tilde{l}_{ij};\ i = 1, \ldots, p;\ j = 1, \ldots, k;\ 1 < m \leq 4$ (usually).
Let further $U = (u_1 \cdots u_k) = (\tilde{L}^T \cdot \tilde{L})^{-1} \tilde{L}^T Q$ (coefficients of the regression of the columns $q_1, \ldots, q_k$ on $\tilde{L}$).

The Promax transformation matrix is then given by

$$\tilde{M} = U \cdot D \qquad \text{where} \qquad D^2 = \text{diag}\{(U^T U)^{-1}\}$$

$\tilde{M}$ is U with normalized columns.

Promax-method

Improves the result of the orthogonal Varimax rotation $\tilde{L} = L \cdot M$ with an oblique transformation $\tilde{\tilde{L}} = \tilde{L} \cdot \tilde{M}$

The loadings should be even closer to 0 or 1 than with Varimax:

Let $Q = (q_{ij}) = (q_1 \cdots q_k)$ with
$q_{ij} = |\tilde{l}_{ij}^{m-1}| \cdot \tilde{l}_{ij};\ i = 1, \ldots, p;\ j = 1, \ldots, k;\ 1 < m \leq 4$ (usually).
Let further $U = (u_1 \cdots u_k) = (\tilde{L}^T \cdot \tilde{L})^{-1} \tilde{L}^T Q$ (coefficients of the regression of the columns $q_1, \ldots, q_k$ on $\tilde{L}$).

The Promax transformation matrix is then given by

$$\tilde{M} = U \cdot D \qquad \text{where} \qquad D^2 = \text{diag}\{(U^T U)^{-1}\}$$

$\tilde{M}$ is U with normalized columns.

factor analysis

rotation methods

Promax-method

Improves the result of the orthogonal Varimax rotation $\tilde{L} = L \cdot M$ with an oblique transformation $\tilde{\tilde{L}} = \tilde{L} \cdot \tilde{M}$

The loadings should be even closer to 0 or 1 than with Varimax:

Let $Q = (q_{ij}) = (q_1 \cdots q_k)$ with
$q_{ij} = |\tilde{l}_{ij}^{m-1}| \cdot \tilde{l}_{ij};\ i = 1, \ldots, p;\ j = 1, \ldots, k;\ 1 < m \leq 4$ (usually).
Let further $U = (u_1 \cdots u_k) = (\tilde{L}^T \cdot \tilde{L})^{-1} \tilde{L}^T Q$ (coefficients of the regression of the columns $q_1, \ldots, q_k$ on $\tilde{L}$).

The Promax transformation matrix is then given by

$$\tilde{M} = U \cdot D \qquad \text{where} \qquad D^2 = \text{diag}\{(U^T U)^{-1}\}$$

$\tilde{M}$ is U with normalized columns.

factor analysis
rotation methods

Promax-method

Improves the result of the orthogonal Varimax rotation $\tilde{L} = L \cdot M$ with an oblique transformation $\tilde{\tilde{L}} = \tilde{L} \cdot \tilde{M}$

The loadings should be even closer to 0 or 1 than with Varimax:

Let $Q = (q_{ij}) = (q_1 \cdots q_k)$ with
$q_{ij} = |\tilde{l}_{ij}^{m-1}| \cdot \tilde{l}_{ij};\ i = 1, \ldots, p;\ j = 1, \ldots, k;\ 1 < m \leq 4$ (usually).
Let further $U = (u_1 \cdots u_k) = (\tilde{L}^T \cdot \tilde{L})^{-1} \tilde{L}^T Q$ (coefficients of the regression of the columns $q_1, \ldots, q_k$ on $\tilde{L}$).

The Promax transformation matrix is then given by

$$\tilde{M} = U \cdot D \qquad \text{where} \qquad D^2 = \text{diag}\{(U^T U)^{-1}\}$$

$\tilde{M}$ is U with normalized columns.

Oblimax-method

The oblique version of the orthogonal Quartimax method.

The variance of the squared loadings within the rows of $\tilde{L}$ should be maximized by variation of the elements of M. M doesn't have to be orthogonal here.

Oblimin-methods

Under this name different oblique methods are summarized that try to minimize the fourth moments of the factor loadings (**Quartimin**, **Covarimin**, **Biquartimin**, ...)

Oblimax-method

The oblique version of the orthogonal Quartimax method.

The variance of the squared loadings within the rows of $\tilde{L}$ should be maximized by variation of the elements of M. M doesn't have to be orthogonal here.

Oblimin-methods

Under this name different oblique methods are summarized that try to minimize the fourth moments of the factor loadings (**Quartimin**, **Covarimin**, **Biquartimin**, ...)

factor analysis

rotation methods

Oblimax-method

The oblique version of the orthogonal Quartimax method.

The variance of the squared loadings within the rows of $\tilde{L}$ should be maximized by variation of the elements of M. M doesn't have to be orthogonal here.

Oblimin-methods

Under this name different oblique methods are summarized that try to minimize the fourth moments of the factor loadings (**Quartimin**, **Covarimin**, **Biquartimin**, ...)

factor analysis

rotation methods

Oblimax-method

The oblique version of the orthogonal Quartimax method.

The variance of the squared loadings within the rows of $\tilde{L}$ should be maximized by variation of the elements of M. M doesn't have to be orthogonal here.

Oblimin-methods

Under this name different oblique methods are summarized that try to minimize the fourth moments of the factor loadings (**Quartimin**, **Covarimin**, **Biquartimin**, ...)

factor analysis

rotation methods

Oblimax-method

The oblique version of the orthogonal Quartimax method.

The variance of the squared loadings within the rows of $\tilde{L}$ should be maximized by variation of the elements of M. M doesn't have to be orthogonal here.

Oblimin-methods

Under this name different oblique methods are summarized that try to minimize the fourth moments of the factor loadings (**Quartimin**, **Covarimin**, **Biquartimin**, ...)