Bayes oracle and asymptotic optimality of multiple testing procedures under sparsity

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Abstract

We investigate asymptotic optimality of a large class of multiple testing rules using the framework of Bayesian Decision Theory. We consider the parametric setup, where the observations come from a normal scale mixture model, and assume a loss which is additive with respect to individual tests. Our model can be used for testing point null hypotheses of no signals (zero effects), as well as to distinguish large signals from a multitude of very small effects. Optimality of a rule is proved by showing that the ratio of its Bayes risk and that of the Bayes oracle (a rule which minimizes the Bayes risk) converges to one within our chosen asymptotic framework. Our main interest is in the asymptotic scheme under which the proportion p of "true" alternatives converges to zero. We fully characterize the class of fixed threshold multiple testing rules which are asymptotically optimal and hence derive conditions for the asymptotic optimality of the rules controlling the Bayesian False Discovery Rate (BFDR). We also provide conditions under which the popular Benjamini-Hochberg procedure is asymptotically optimal and show that for a wide class of sparsity levels, its threshold can be approximated very well by a non-random threshold. Our results extend to more general priors as well as to model selection in a linear regression setting under orthogonality.