The Generalized Normal Distribution

Related to

Generalized Fisher's *entropy type* Information

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Abstract

Working on the generalization of Fisher's entropy type Information measure (say $J_a(X)$, X a rv with a an extra parameter), following the well-known Vajda parametric measure of information, we came across two new results:

(i) the classical entropy inequality was still true, for the introduced generalized entropy power (say $N_a(X)$), ie $J_a(X)N_a(X)>=p$ invariant under dilations and

(ii) (ii) Blachan-Stam inequality was still valid.

Moreover adopting the Gross logarithmic inequality and its equivalent Logarithm Sobolev Inequality (LSI) form, considering when the corresponding inequality is optimal, that was true when the involved function takes a particular form, which for us was "similar" to multivariate Normal distribution, with an extra parameter: we called the emerged from the LSI distribution, generalized (or gamma-order) Normal distribution, with three parameters (position, scale, shape). It took some time to obtain their MLE, and is a rather complicated in practice. But it is useful that it serves as the Gaussian in the Information theory, for the introduced generalized entropy type information measures. Moreover for different values of the extra (shape) parameter coincides with Dirac, Uniform, Normal, Laplace distributions, and offers a number of other extensions to the well- known distributions: truncated, lognormal among others.

References

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