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Statistical Methods

Tutorial in WS 2017/18 for Monday, 11.12.17
from 12:00-13:30 in S2 053

Tutorial 5

Evaluation of point estimators

Exercise 25 (Unbiased estimators)
Let T_{1n} and T_{2n} be two unbiased estimators for θ.

a) Show that $T_n = \alpha T_{1n} + (1-\alpha)T_{2n}$ is an unbiased estimator for θ for all $\alpha \in [0,1]$.

b) Suppose that T_{1n} and T_{2n} are uncorrelated and that $\mathrm{Var}(T_{1n}) = \sigma_{1n}^2$ and $\mathrm{Var}(T_{2n}) = \sigma_{2n}^2$. Calculate $\mathrm{Var}(T_n)$.

c) Find the value of $\alpha \in [0,1]$ such that $\mathrm{Var}(T_n)$ is minimal.

Exercise 26 (MLE, ME and Fisher information)
Let Y_i for $i = 1, ..., n$ be iid random variables with pdf given by

$$f(y;b) = \frac{2}{b} y \exp\left(-\frac{y^2}{b}\right) \mathbb{1}_{(0,\infty)}(y), \qquad b > 0.$$

a) Calculate the Fisher information.

b) Derive the MLE for b.

c) Derive two MEs for b, one based on the mean and one on the second moment.

d) Are the three estimators unbiased?

e) Calculate the MSE of the MLE and of one of the MEs and comment on the result.

f) Is the best unbiased estimator unique?

Exercise 27 (MVUE and Cramér-Rao bound)
Let $X \sim Bin(n,p)$ be a binomial distribution with known $n \in \mathbb{N}$. Is

$$T(X) := \frac{X(n-X)}{n(n-1)}$$

a minimum variance unbiased estimator for $\gamma := p(1-p)$?

Exercise 28 (Unbiased estimators and MLE)
Suppose that the random variables Y_i satisfy

$$Y_i = \beta x_i + \epsilon_i,$$

where x_i are fixed constants and ϵ_i are iid $N(0, \sigma^2)$-distributed random variables for $i = 1, ..., n$ with unknown σ^2.

a) Find a two-dimensional sufficient statistic for (β, σ^2).

b) Find the MLE of β and show that it is an unbiased estimator of β.

c) Find the distribution of the MLE of β.

d) Show that $\sum Y_i / \sum x_i$ is an unbiased estimator of β

e) Calculate the variance of $\sum Y_i / \sum x_i$ and compare it to the variance of the MLE.

Exercise 29 (Best unbiased estimator)
Let X_i for $i = 1, ..., n$ be a random sample from a population with pdf

$$f(x; \theta) = \frac{1}{2\theta}$$

for $-\theta < x < \theta$ and $\theta > 0$. Find the best unbiased estimator of θ if one exists.

Exercise 30 (Consistent estimator)
A random sample X_i for $i = 1, ..., n$ is drawn from a population with pdf

$$f(x; \theta) = \frac{1}{2}(1 + \theta x)$$

for $-1 < x < 1$ and $-1 < \theta < 1$. Find a consistent estimator of θ.