JOHANNES KEPLER
UNIVERSITY LINZ

# Workshop on Numerical Analysis of Stochastic Partial Differential Equations (NASPDE) 2017

June 21-22, 2017

Johannes Kepler University, Linz

Institute for Stochastics and
Doctoral Program "Computational Mathematics",
Johannes Kepler University, Linz

Doctoral Program
Computational Mathematics
Numerical Analysis and Symbolic Computation

FWF
Der Wissenschaftsfonds.

LAND
OBERÖSTERREICH

# Contents

# Information

Dear participant of the NASPDE workshop 2017,

It is our pleasure to welcome you in Linz at the **Johannes Kepler University Linz** for the workshop on Numerical Analysis of Stochastic Partial Differential Equations (NASPDE) 2017 hosted by the Institute for Stochastics and the Doctoral Program "Computational Mathematics".

The goal of this NASPDE 2017 workshop is to promote recent advances in numerical analysis related to Stochastic Partial Differential Equations (SPDEs) and Random Processes and their applications. It covers various domains from simulation and analysis of numerical methods for PDEs or SPDEs, to propagation of uncertainty and the development of efficient simulation methods for random processes.

The NASPDE workshop takes place once a year. The previous meetings were held in Gothenburg 2016, Inria Sophia Antipolis 2015, Lausanne 2014, Rennes 2013, Warwick 2012, Freiberg 2010, Edinburgh 2009, Zrich 2008, and Manchester 2007.

This year's host university - the Johannes Kepler University (JKU) Linz – boasts future-oriented academic degree programs, excellence in teaching and research, numerous partnerships in Austria and abroad, and a unique campus with park-like grounds. Although a young university (established 1966), in a short period of time the JKU has become a cutting-edge institution for science, academics, business and the community. Over 19,000 students are enrolled in over 60 modern, hands-on academic degree programs that have outstanding career prospects. Research conducted at the faculties and institutes is recognized worldwide.

We are grateful to the Doctoral Program "Computational Mathematical" whose financial support has made the NASPDE 2017 possible.

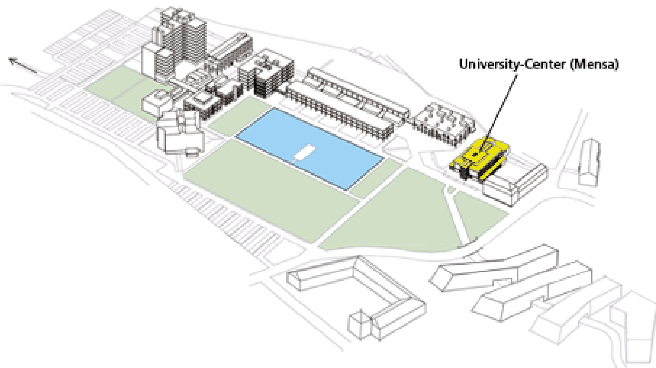
We hope you will enjoy the scientific programme, and take the chance to exchange ideas with colleagues that are also working in the field of numerical analysis of stochastic differential equations!

Evelyn Buckwar and Andreas Thalhammer
*Organizing Committee*

## Venue

The campus is located in the northeast of Linz and can be reached via public transportation: Take Tram 1 or 2 in the direction of "Universität" from the main train station in Linz. Duration: approx. 25 min.

The workshop takes place at the **University-Center (Mensa)** Linz on the 1st floor in the room **Festsaal B**.

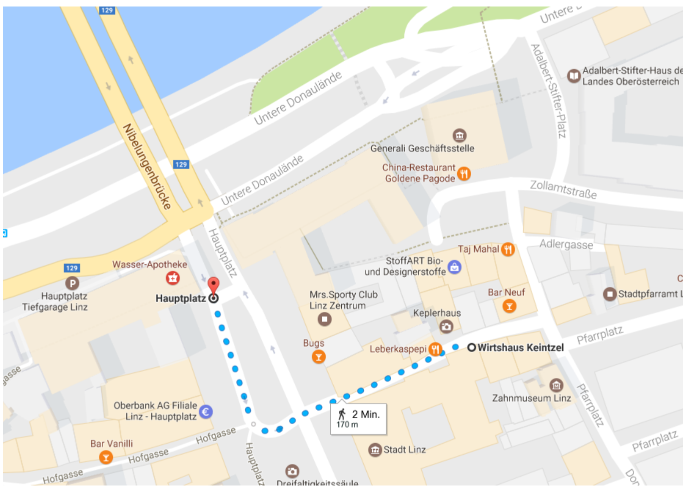



## Workshop dinner

The workshop dinner takes place on Wednesday, June 21 at 19:00 at

Gasthaus Keintzel
Rathausgasse 6-8
A-4020 Linz.

To get to the city center take Tram 1 (in direction Auwiesen) or Tram 2 (in direction Solar City) to the tram station "Hauptplatz" and following the map below.



# Abstracts

## Simulation of infinite-dimensional Lévy-Processes

Andrea Barth
University of Stuttgart

Abstract: In various applications, stochastic partial differential equations are not driven by Gaussian noise but rather by one whose marginal distributions have heavier tails. Unlike the case of an infinite-dimensional Gaussian process, a general infinite-dimensional Lévy process cannot be built from independent, one-dimensional Lévy processes and still admit these one-dimensional distributions as its marginals. In this talk I introduce an approach to construct time-dependent random fields that have marginal distributions which follow certain Lévy measures. I show convergence of the method and wrap up with some numerical examples.

This is joint work with Andreas Stein (University of Stuttgart).

---

## A fully discrete approximation of the one-dimensional stochastic heat equation

David Cohen
University of Umeå

Abstract: A fully discrete approximation of one-dimensional nonlinear stochastic heat equations driven by multiplicative noise is presented. A standard finite difference approximation is used in space and a stochastic exponential method is used for the temporal approximation. This explicit time integrator allows for error bounds in $L^p(\Omega)$, uniformly in time and space. Furthermore, uniform almost sure convergence of the numerical solution is proved. Numerical experiments are presented and confirm the theoretical results.

The presentation is based on an ongoing joint work with Rikard Anton (Umeå University) and Lluis Quer-Sardanyons (Universitat Autonoma de Barcelona).

---

## Numerical computation about metastable states of phase separation models

Ludovic Goudenège
CNRS

Abstract: For many phase separation models, there is a huge literature about computation of deterministic stable and unstable states. In a probabilistic setting and under some regime -for instance in the large deviation asymptotic- these states play an important role in the computation of quantities of interest. Maybe the most important are the expected exit times of region near metastable states. In this talk I will present estimator, schemes and ideas about numerical computation of these quantities. Precisely I will first present the generalization of adaptive multilevel splitting algorithm in a general framework. Then I will explain how to use it to create an estimator of expected exit times or expected length of reactive trajectories. Finally I will present some numerical result in the context of phase separation models.

---

## Efficient time discretisation of parabolic SPDEs

Gabriel Lord
Heriot Watt University Edinburgh

Abstract: We introduce adaptive time stepping techniques to control growth in the numerical solution of SPDEs. This can be thought of as an alternative to proving moment bounds for the numerical method and to using a fixed step taming method. Ideas and the convergence result will be illustrated with some numerical experiments that also show that the adaptivity leads to more accurate solutions.

If time permits we will introduce a new exponential based method for time stepping for SPDEs with multiplicative noise which have an improved rate of convergence in specific circumstances.

---

## An adaptive algorithm for PDE problems with random data

David Silvester
University of Manchester

Abstract: We present a new adaptive algorithm for computing stochastic Galerkin finite element approximations for a class of elliptic PDE problems with random data. Specifically, we assume that the underlying differential operator has affine dependence on a large, possibly infinite, number of random parameters. Stochastic Galerkin approximations are then sought in the tensor product space $X \otimes \mathcal{P}$, where $X$ is a finite element space associated with a physical domain and $\mathcal{P}$ is a set of multivariate polynomials over a finite-dimensional manifold in the (stochastic) parameter space.

Our adaptive strategy is based on computing two error estimators (the spatial estimator and the stochastic one) that reflect the two distinct sources of discretisation error and, at the same time, provide effective estimates of the error reduction for the corresponding enhanced approximations. In particular, our algorithm adaptively 'builds' a polynomial space over a low-dimensional manifold in the infinitely-dimensional parameter space such that the discretisation error is reduced most efficiently (in the energy norm). Convergence of the adaptive algorithm is demonstrated numerically.

This is joint work with Alex Bespalov (University of Birmingham) and Catherine Powell (University of Manchester)

---

## $L^p$–estimates and regularity for SPDEs with monotone semilinearity

David Siska

University of Edinburgh

Abstract: We prove $L^p$-estimates for semilinear stochastic partial differential equations (on bounded domains) with monotone semilinear term. These are used together with known results for linear SPDEs to obtain regularity results for such equations. These are interior estimates in Sobolev norms and estimates up to the boundary in weighted Sobolev spaces. This is joint work with Neelima.

---

## Quasi- and Multilevel Monte Carlo Methods for Computing Posterior Expectations

Aretha Teckentrup

University of Edinburgh

Abstract: The parameters in mathematical models for physical processes are often impossible to determine fully or accurately, and are hence subject to uncertainty. By modelling the input parameters as stochastic processes, it is possible to quantify the induced uncertainty in the model outputs. Based on available information, a prior distribution is assigned to the input parameters. If in addition some dynamic data (or observations) related to the model outputs are available, a better representation of the parameters can be obtained by conditioning the prior distribution on these data, leading to the posterior distribution in the Bayesian framework.

In most situations, the posterior distribution is intractable in the sense that the normalising constant is unknown and exact sampling is unavailable. Using Bayes Theorem, we show that posterior expectations of quantities of interest can be written as the ratio of two prior expectations involving the quantity of interest and the likelihood of the observed data. These prior expectations can then be computed using Quasi-Monte Carlo and multilevel Monte Carlo methods.

In this talk, we give a convergence and complexity analysis of the resulting ratio estimators, and demonstrate their effectiveness on a typical model problem in uncertainty quantification.

---

## Uncertainty Quantification for moving boundary problems

Michael Tretyakov
University of Nottingham

Abstract: The main motivation for the considered UQ problem comes from modelling of one of the main manufacturing processes for producing advanced composites – resin transfer molding (RTM). We consider one-dimensional and two-dimensional models of the stochastic resin transfer molding process,

which are formulated as random moving boundary problems. We study their properties, analytically in the one-dimensional case and numerically in the two-dimensional case. We show how variability of time to fill depends on correlation lengths and smoothness of a random permeability field.

We will also briefly discuss experimental results for fibre preforms manufactured with Automated Dry Fibre Placement (ADFP) in a laboratory as well as Bayesian inversion for RTM. The talk is based on works with Minho Park, Marco Iglesias, Mikhail Matveev, Arthur Jones, Andy Long, Frank Ball.

---

# Posters

The poster session takes place on Wednesday, June 21, at 15:10 in parallel to the welcome reception.

*Full discretization scheme for nonlinear SPDEs with locally monotone coefficients*
**Neelima Neelima**

*Numerical solution of fractional elliptic equations driven by spatial white noise*
**Kristin Kirchner**

*Applied Samples for SPDEs in Finance and Markets*
**Albasher Shareif**

*Importance Sampling Techniques for Stochastic Partial Differential Equations*
**Andreas Thalhammer**

# Time table

## Wednesday, June 21

| | | |
|---|---|---|
| 09:00 – 09:30 | Registration | |
| 09:30 – 09:45 | Opening | |
| SESSION 1 – Chair: David Siska | | |
| 09:45 – 10:35 | *David Cohen (University of Umeå)* | A fully discrete approximation of the one-dimensional stochastic heat equation |
| 10:35 – 11:30 | *Gabriel Lord (Heriott Watt University)* | Efficient time discretisation of parabolic SPDEs |
| LUNCH BREAK | | |
| SESSION 2 – Chair: David Silvester | | |
| 13:30 – 14:20 | *Michael Tretyakov (University of Nottingham)* | Uncertainty Quantification for moving boundary problems |
| 14:20 – 15:10 | *Aretha Teckentrup (University of Edinburgh)* | Quasi- and Multilevel Monte Carlo Methods for Computing Posterior Expectations |
| 15:10 – 16:30 | Poster session | Coffee break & welcome reception |
| EVENING BREAK | | |
| 19:00 – open end | Workshop dinner | |

## Thursday, June 22

| | | |
|---|---|---|
| SESSION 3 – Chair: Aretha Teckentrup | | |
| 09:00 – 09:50 | *David Silvester* *(University of Manchester)* | An adaptive algorithm for PDE problems with random data |
| 09:50 – 10:40 | *Ludovic Goudenège* *(CNRS)* | Numerical computation about metastable states of phase separation models |
| COFFEE BREAK | | |
| SESSION 4 – Chair: David Cohen | | |
| 11:10 – 12:00 | *Andrea Barth* *(University of Stuttgart)* | Simulation of infinite-dimensional Lévy processes |
| 12:00 – 12:50 | *David Siska* *(University of Edinburgh)* | $L^p$-estimates and regularity for SPDEs with monotone semilinearity |
| 12:50 – 13:00 | Closing | |